



Aitchison College Academic Resources.

IGCSE MATHEMATICS (0580) RESOURCES

This document includes complete notes of each unit with unsolved and solved past paper questions. Don't forget to go through it and we hope this document will help you preparing for your exams.
Best of Luck!



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Paper Pattern:

Extended assessment

Extended candidates take Paper 2 and Paper 4. The questions are based on the Extended subject content only:

Paper 2: Non-calculator (Extended)

2 hours
100 marks 50%
Structured and unstructured questions
Use of a calculator is **not** allowed
Externally assessed

Paper 4: Calculator (Extended)

2 hours
100 marks 50%
Structured and unstructured questions
A scientific calculator is required
Externally assessed

Information on availability is in the **Before you start** section.

Specimen Paper:

[Specimen-paper-2](#)

[Specimen paper 4](#)



Formulae Sheet:

Area, A , of triangle, base b , height h .

$$A = \frac{1}{2}bh$$

Area, A , of circle of radius r .

$$A = \pi r^2$$

Circumference, C , of circle of radius r .

$$C = 2\pi r$$

Curved surface area, A , of cylinder of radius r , height h .

$$A = 2\pi rh$$

Curved surface area, A , of cone of radius r , sloping edge l .

$$A = \pi rl$$

Surface area, A , of sphere of radius r .

$$A = 4\pi r^2$$

Volume, V , of prism, cross-sectional area A , length l .

$$V = Al$$

Volume, V , of pyramid, base area A , height h .

$$V = \frac{1}{3}Ah$$

Volume, V , of cylinder of radius r , height h .

$$V = \pi r^2 h$$

Volume, V , of cone of radius r , height h .

$$V = \frac{1}{3}\pi r^2 h$$

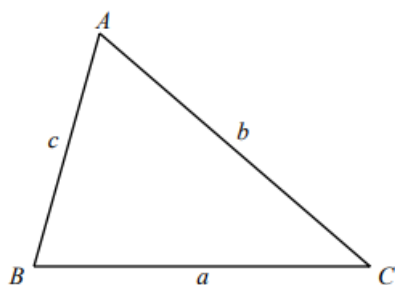
Volume, V , of sphere of radius r .

$$V = \frac{4}{3}\pi r^3$$

For the equation $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the triangle shown,



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area} = \frac{1}{2}ab \sin C$$



Unit 1: Number

1 Number

E1.1 Types of number

Identify and use:

- natural numbers
- integers (positive, zero and negative)
- prime numbers
- square numbers
- cube numbers
- common factors
- common multiples
- rational and irrational numbers
- reciprocals.

Notes and examples

Example tasks include:

- convert between numbers and words, e.g.
six billion is 6 000 000 000
10 007 is ten thousand and seven
- express 72 as a product of its prime factors
- find the highest common factor (HCF) of two numbers
- find the lowest common multiple (LCM) of two numbers.

E1.2 Sets

Understand and use set language, notation and Venn diagrams to describe sets and represent relationships between sets.

Notes and examples

Venn diagrams are limited to two or three sets.

The following set notation will be used:

- $n(A)$ Number of elements in set A
- $\in$ "... is an element of ..."
- $\notin$ "... is not an element of ..."
- A' Complement of set A
- $\emptyset$ The empty set
- $\mathcal{U}$ Universal set
- $A \subseteq B$ A is a subset of B
- $A \not\subseteq B$ A is not a subset of B
- $A \cup B$ Union of A and B
- $A \cap B$ Intersection of A and B .

Example definition of sets:

$A = \{x: x \text{ is a natural number}\}$

$B = \{(x, y): y = mx + c\}$

$C = \{x: a \leq x \leq b\}$

$D = \{a, b, c, \dots\}$.

E1.3 Powers and roots

Calculate with the following:

- squares
- square roots
- cubes
- cube roots
- other powers and roots of numbers.

Notes and examples

Includes recall of squares and their corresponding roots from 1 to 15, and recall of cubes and their corresponding roots of 1, 2, 3, 4, 5 and 10, e.g.:

- Write down the value of $\sqrt{169}$.
- Work out $5^2 \times \sqrt[3]{8}$.



E1.4 Fractions, decimals and percentages

Notes and examples

1 Use the language and notation of the following in appropriate contexts:

- proper fractions
- improper fractions
- mixed numbers
- decimals
- percentages.

Candidates are expected to be able to write fractions in their simplest form.

Recurring decimal notation **is** required, e.g.

- $0.1\dot{7} = 0.1777\dots$
- $0.\dot{1}2\dot{3} = 0.1232323\dots$
- $0.123 = 0.123123\dots$

2 Recognise equivalence and convert between these forms.

Includes converting between recurring decimals and fractions and vice versa, e.g. write $0.1\dot{7}$ as a fraction.

E1.5 Ordering

Notes and examples

Order quantities by magnitude and demonstrate familiarity with the symbols $=$, $\neq$, $>$, $<$, $\geq$ and $\leq$.

E1.6 The four operations

Notes and examples

Use the four operations for calculations with integers, fractions and decimals, including correct ordering of operations and use of brackets.

Includes:

- negative numbers
- improper fractions
- mixed numbers
- practical situations, e.g. temperature changes.

E1.7 Indices I

Notes and examples

1 Understand and use indices (positive, zero, negative, and fractional).

Examples include:

- $6^{\frac{1}{2}} = \sqrt{6}$
- $16^{\frac{1}{4}} = \sqrt[4]{16}$
- find the value of 7^{-2} , $81^{\frac{1}{2}}$, $8^{-\frac{2}{3}}$.

2 Understand and use the rules of indices.

e.g. find the value of $2^{-3} \times 2^4$, $(2^3)^2$, $2^3 \div 2^4$.

E1.8 Standard form

Notes and examples

1 Use the standard form $A \times 10^n$ where n is a positive or negative integer and $1 \leq A < 10$.

2 Convert numbers into and out of standard form.

3 Calculate with values in standard form.



E1.9 Estimation	Notes and examples
1 Round values to a specified degree of accuracy. 2 Make estimates for calculations involving numbers, quantities and measurements. 3 Round answers to a reasonable degree of accuracy in the context of a given problem.	Includes decimal places and significant figures. e.g. write 5764 correct to the nearest thousand. e.g. by writing each number correct to 1 significant figure, estimate the value of $\frac{41.3}{9.79 \times 0.765}$
E1.10 Limits of accuracy	Notes and examples
1 Give upper and lower bounds for data rounded to a specified accuracy. 2 Find upper and lower bounds of the results of calculations which have used data rounded to a specified accuracy.	e.g. write down the upper bound of a length measured correct to the nearest metre. Example calculations include: • calculate the upper bound of the perimeter or the area of a rectangle given dimensions measured to the nearest centimetre • find the lower bound of the speed given rounded values of distance and time.
E1.11 Ratio and proportion	Notes and examples
Understand and use ratio and proportion to: • give ratios in their simplest form • divide a quantity in a given ratio • use proportional reasoning and ratios in context.	e.g. 20:30:40 in its simplest form is 2:3:4. e.g. adapt recipes; use map scales; determine best value.



E1.15 Time	Notes and examples
1 Calculate with time: seconds (s), minutes (min), hours (h), days, weeks, months, years, including the relationship between units.	1 year = 365 days.
2 Calculate times in terms of the 24-hour and 12-hour clock.	In the 24-hour clock, for example, 3.15 a.m. will be denoted by 03 15 and 3.15 p.m. by 15 15.
3 Read clocks and timetables.	Includes problems involving time zones, local times and time differences.
E1.16 Money	Notes and examples
1 Calculate with money.	
2 Convert from one currency to another.	
E1.17 Exponential growth and decay	Notes and examples
Use exponential growth and decay.	e.g. depreciation, population change. Knowledge of e is not required.
E1.18 Surds	Notes and examples
1 Understand and use surds, including simplifying expressions.	Examples include: • $\sqrt{20} = 2\sqrt{5}$ • $\sqrt{200} - \sqrt{32} = 6\sqrt{2}$.
2 Rationalise the denominator.	Examples include: • $\frac{10}{\sqrt{5}} = 2\sqrt{5}$ • $\frac{1}{-1+\sqrt{3}} = \frac{1+\sqrt{3}}{2}$.



Types of Number

Expressing Numbers

Numbers can be expressed in different ways. They can be represented numerically, e.g. 600, or in word form, e.g. six hundred. To convert from numerical form to word form, the non-zero digits of the places must be noted down and converted to their respective places' word form.

For example, 61,520 can be written as sixty-one thousand, five hundred and twenty. The six is in the ten-thousands' place, the one is in the thousands' place, the five is in the hundreds' place, and the two is in the tens' place. Since zero is in the units' place, it is ignored.

Similarly, 509,007 can be written as five hundred and nine thousand, and seven. Note that each three digits starting from the right are collected into units, tens and hundreds.

The first three digits are written as they are, the next three digits are followed by the word 'thousand', the next three are followed by the word 'million', then 'billion', 'trillion', and so on. This method can also be used to convert words into numerical form.

For example, "six hundred-billion" means that there are $3 + 3 + 3 + 3 = 12$ digits in that number. So it can be written down as 600,000,000,000, or 6 followed by eleven 0s.

Broad Sets of Numbers

Real Numbers: These are the set of all numbers that are dealt with at IGCSE Level. This excludes imaginary numbers. The symbol $\mathbb{R}$ may be used to represent this set.

Integers: These are whole numbers, i.e. numbers without any fraction/decimal. They can be positive, negative, or zero. The symbol $\mathbb{Z}$ may be used to represent integers.

Natural Numbers: These are the set of all positive integers, e.g. 1, 2, 3, ... The symbol $\mathbb{N}$ may be used to denote natural numbers.

Rational and Irrational Numbers: Rational numbers include any real numbers that can be expressed as a fraction in the form of a/b , where a and b are whole numbers. Irrational numbers cannot be expressed in this way. All terminating and recurring decimals are rational. π is irrational; this is a constant that will be used in future chapters.



Other Types of Number

Prime Numbers: These are numbers that have two factors only, one of which is 1, and the second of which is the number itself. All primes are odd, except for 2. The first ten primes are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. 1 is not prime since it only has one factor, which is itself.

Square Numbers: These are numbers derived by multiplying a positive integer by itself. For example, $4 * 4 = 16$, so 16 is a square. 4 itself is a square, as it is the product of 2 and itself.

Cube Numbers: These are numbers derived by multiplying a positive integer by itself, twice. For example, $3 * 3 * 3 = 27$, so 27 is a cube.

Reciprocals: The reciprocal of any number is the result of dividing 1 by that number. For example, the reciprocal of 2 is $1/2$. For a real number, n , the reciprocal is $1/n$.

Using Prime Factors

Prime factors: These are prime numbers that, when multiplied together, give a product of a composite (non-prime) number. For example, the prime factors of 6 are 2 and 3. The prime factors of 12 are 2, 2, and 3.

To find the prime factors of any composite number, a factor tree can be used. This involves splitting up a number into factors, and splitting up each of those factors until all of them lead to primes. These cannot be split into anything but 1 and themselves.

If asked to write prime factors of a number with powers, the following example may be used: $72 = (2*2*2) * (3*3)$.

If all the unique prime factors of a number appear an even number of times, the number is a square. For example, $(2*2*2*2) * (3*3) = 144$, which is the square of 12. If all the prime factors appear a number of times which is any multiple of 3, the number is a cube. For example, $(5*5*5) * (2*2*2*2*2*2) = 8000$, which is the cube of 20.

Highest Common Factor: To find common factors, all the factors of two or more numbers can be listed down. Factors appearing in all these lists are common. The highest of these is known as the HCF, or highest common factor. This is sometimes referred to as the GCD, or greatest common denominator, as well. To find the HCF more efficiently, a number can be decomposed into its prime factors, and then the common prime factors can be multiplied to find the HCF.

Lowest Common Multiple: To find common multiples, the first few multiples of two or more numbers can be listed down. Multiples appearing in all lists are



common. The lowest of these is known as the LCM, or lowest common multiple. To find the LCM more efficiently, each number can be again broken down into the product of its prime factors. Then, the prime factors of one of the numbers that are NOT prime factors of the other numbers should be taken out. These should be multiplied by the originally chosen number. This yields the same value regardless of which number is originally chosen, and this is the LCM.

Sets

Sets are collections of elements

Elements in a set can be any individual piece of information, numbers, letters, coordinates, etc.

A set can be defined by writing its elements inside of '{ }' brackets. E.g. {1, 2, 4, 8} is the set of the factors of 8

If the elements in a set follow a rule, a colon can be used to separate the general type of element from the rule. E.g. in the set $\{x: x^2 < 50\}$, the elements are the numbers that are less than fifty when squared. Coordinates and their corresponding equations can also be represented this way, e.g. $\{(x, y) : y = 3x - 2\}$ defines the points that lie on the line $y = 3x - 2$

Set Representation

Uppercase letters (e.g. A, B, C) usually represent sets, whereas lowercase letters (e.g. a, b, c) represent general elements

The symbol ϵ or U denotes the universal set (set of everything). E.g. if the subject is the factors of 12, $\epsilon = \{1, 2, 3, 4, 6, 12\}$

The symbol $\emptyset$ denotes the empty set (with no elements). E.g. $\{x : x \text{ is an even prime bigger than } 2\} = \emptyset$ as there are no even primes bigger than 2

Algebraic Operations of Sets

$n(A)$ represents the number of elements in the set A. E.g. $n\{-1, 0, 1\} = 3$

$a \in A$ means a is an element of A. E.g. if E is the set of even numbers, $24 \in E$

$A \subset B$ means set A is a proper subset of a larger set B. E.g. if $X = \{1, 2, 3\}$ and $Y = \{0, 1, 2, 3, \dots\}$, then $X \subset Y$

$A \subseteq B$ means that set A is a subset of set B, which has the same elements, but does not necessarily have to be larger. E.g. if $N = \{2, 4, 6\}$, then $\{2, 4, 6\} \subseteq N$

Putting a cross through the $\in$, $\subset$, or $\subseteq$ signs renders them untrue, similar to the $=$ sign. So $a \notin A$, for example, means that a is not an element of A



If $A \subset B$, then $A \subseteq B$, but if $A \subseteq B$, then A does not necessarily have to be a proper subset of B

$A \cap B$ represents the intersection of A and B . The intersection of two sets refers to the elements that are common among both sets. E.g. if $A = \{1, 2, 3\}$, and $B = \{2, 4, 6\}$, then $A \cap B = \{2\}$

$A \cup B$ represents the union of A and B . The union of two sets refers to all of the elements in either or both sets. E.g. if $A = \{1, 4, 9\}$ and $B = \{1, 3, 5\}$, then $A \cup B = \{1, 3, 4, 5, 9\}$

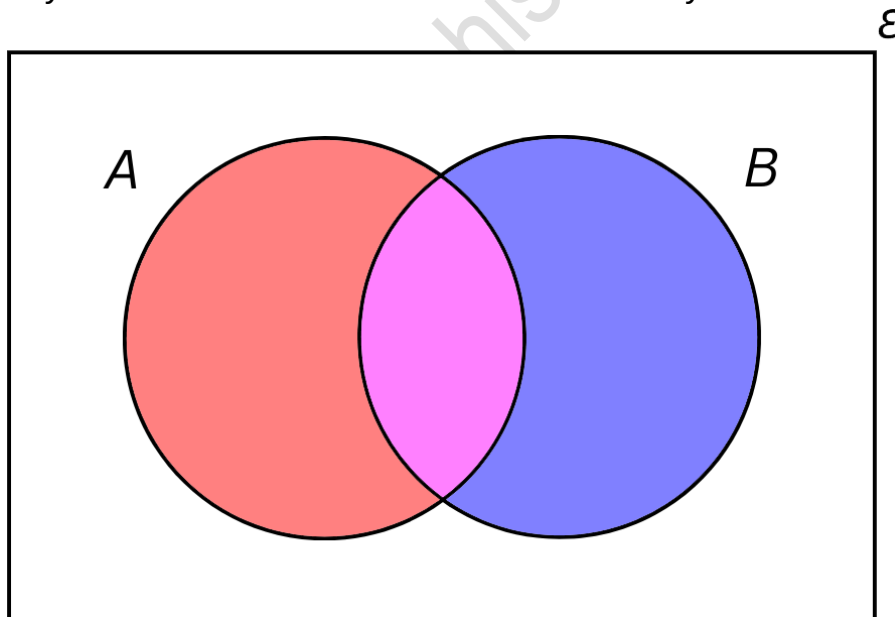
A' represents the complement of A . This refers to all the elements excluding those in set A . E.g. if ϵ denotes $\{1, 2, 3, \dots, 20\}$, and $A = \{1, 4, 9, 16\}$, then $A' = \{2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20\}$

In any case, $\epsilon' = \emptyset$, as the complement of ALL the elements is taken, which leaves no elements

Venn Diagrams

A Venn diagram is a way to illustrate all the elements within sets, as well as any intersections amongst them

Venn diagrams usually have a rectangle to illustrate ϵ , and circles to represent smaller sets. These circles may overlap depending on whether the sets have any common elements. All circles are entirely inside the rectangle



In the above Venn diagram, ϵ is represented by the entire rectangle, set A is represented by the red and pink area, and set B is represented by the blue and pink area



Since the pink area is common to both sets A and B, we can say that the pink area is $A \cap B$

$A \cup B$ is represented by the red, blue and pink areas

A' is represented by the white and blue areas, and B' is represented by the white and red areas

$A' \cap B$ is represented by the blue area only, and $B' \cap A$ is represented by the red area only. $A' \cap B'$ is depicted by the white area only

$A' \cup B = A'$, and $B' \cup A = B'$. $A' \cup B'$ is shown by all the area except for the pink area

Powers and Roots

Powers

The power of a number signifies how many times a number is multiplied by itself

For a^b , a is the base and b is the power of a. a is multiplied by itself b-1 times
 6^2 means multiplying 6 with itself once, which is $6*6 = 36$

Similarly, 2^4 means multiplying 2 with itself thrice, which is $2*2*2*2 = 16$

The power is also called the index (plural: indices) or exponent

Any number raised to the power of 1 is itself, since it is not multiplied with itself any number of times; e.g. $5^1 = 5$

A negative power indicates the reciprocal of the number had it been raised to the positive power. So 3^{-2} is just $1/3^2 = 1/(3*3) = 1/9$

For two terms, n^a and n^b , $n^a * n^b = n^{a+b}$. Similarly, $n^a/n^b = n^{a-b}$.

n^0 can be written as n^{a-a} where a can be any non-zero number. This can be simplified to $n^a/n^a = 1$. Hence, $n^0 = 1$ for any real number, n.

Roots

Roots are functions that act inversely to powers

A square root of a number is another number that when squared, equals the original number; e.g. the square root of 25 = 5, as $5^2 = 25$

The symbol ' $\sqrt{\quad}$ ' depicts the square root

If another number is written before the root symbol, it means the root of that respective power. So $\sqrt[3]{\quad}$ means the cube root. $\sqrt[3]{27}$, for example, is 3; this is because $3^3 = 27$

Roots can be depicted as reciprocals of the original powers. So $\sqrt{2}$ is just $2^{1/2}$

Note that a square root always returns a positive value, but when solving for $x^2 = y$, both the positive and negative values of the root of y are considered. This applies to all even powers for x



The following table shows some more complex properties of powers and roots:

Property	Example
$(a^m)^n = a^{mn}$	$(3^3)^2 = (3*3*3)*(3*3*3) = 3^6 = 3^{3*2}$
$a^{m/n} = \sqrt[n]{a^m}$	$3^{3/2} = 3^{3 * 1/2} = (3^3)^{1/2} = \sqrt{3^3}$
$(a/b)^n = a^n/b^n$	$(1/3)^2 = (1/3)*(1/3) = (1*1)/(3*3) = 1^2/3^2$
$(a/b)^{-n} = b^n/a^n$	$(5/6)^{-1} = 1/(5/6) = 6/5 = (6/5)^1$

Topic: Limits of Accuracy (1.10)

Major Terms and associated formulas:

Error:

Range of numerical where actual value lies.

Formula:

$$\text{Error} = \text{Scale Value} / 2$$

***Scale value:** The value which is physically readable on scale.

Solved example:

Q: A rectangle has length 10 cm and width 6cm. Both measurement are nearest to cm. Find the error.

Solution:

***Nearest to cm = 1cm = Scale value**

$$\frac{1}{2} = 0.5 \text{ cm}$$

$$\text{Error} = 0.5 \text{ cm}$$

Upper Bound:

It means how high a number could be before rounded

Formula:

$$\text{Upper bound} = \text{Error} + \text{Actual value}$$

Lower Bound:

It means how low a number could be before rounded

Formula:

$$\text{Lower bound} = \text{Actual value} - \text{Error}$$

Solved example:

Q: A rectangle has length 10 cm and width 6cm. Both measurement are



nearest to cm. Find the upper and lower bound of both measurements.

Solution:

***First we would find the error**

$$\frac{1}{2} = 0.5 \text{ cm}$$

$$\text{Error} = 0.5 \text{ cm}$$

***Now apply above formula of upper and lower bound.**

Upper Bound = $10 + 0.5$	$= 6 + 0.5$
$= 10.5 \text{ cm}$	$= 6.5 \text{ cm}$
Lower Bound = $10 - 0.5$	$= 6 - 0.5$
$= 9.5 \text{ cm}$	$= 5.5 \text{ cm}$

Rules of using upper and lower bound in multiplication or division:

Multiplying two numbers together: $T = a \times b$

The **upper bound of T** can be found by multiplying together the **upper bound of a** and the **upper bound of b**.

The **lower bound of T** can be found by multiplying together the **lower bound of a** and the **lower bound of b**.

Dividing a number by another number: $T = a \div b$

The **upper bound of T** can be found by using the **upper bound of a** and dividing it by the **lower bound of b**

The **lower bound of T** can be found by using the **lower bound of a** and dividing it by the **upper bound of b**



Solved Past Paper Question:

(b) Anna's journey home is 47 km, correct to the nearest kilometre. $1/2 = 0.5$
 One day her journey home takes 65 minutes, correct to the nearest 5 minutes. $5/2 = 2.5$
 Calculate the upper bound of her average speed, in km/h, for the journey home.

Distance = 47 km $\begin{cases} 47.5 \text{ km} \\ 46.5 \text{ km} \end{cases}$ $\uparrow S = \frac{D \uparrow}{T \downarrow}$ $\frac{47.5}{62.5}$
 Time = 65 $\begin{cases} 67.5 \text{ min} \\ 62.5 \text{ min} \end{cases}$ $\frac{62.5}{60}$

Answer 45.6 km/h [3]

Topic: Ratio and Proportion (1.11)

Major Terms and associated formulas:

Ratio:

It is a way of comparing one part of a whole with another.

Example: If a cake were sliced into 8 pieces, and shared in the ratio 6:2, this means that **person A receives 6 pieces, and person B receives 2 pieces.**

Simplifying Ratio:

Let's consider the first given example.

The ratio is 6:2

We could also express this ratio as **3:1**, even though we still have 8 pieces.

Both sides of the ratio have been divided by 2

The amount that each person gets relative to the other is still the same.

In this case, person A receives 3 times as much as person B.

Using Ratio to find fraction:

Let's consider the first given example.

The ratio is 6:2

Person A receives 6 pieces out of 8, or **6 / 8** of the cake

This could be simplified to of the pizza **3 / 4**

Person B received 2 slices out of 8, or **2 / 8** of the cake

This could be simplified to **1 / 4** of the pizza.

Combining 2 ratios into 3-part ratios:

Example: A farm has 85 animals



Ratio of cow to sheep 2:3

Ratio of sheep to goat 6:7

We need to find a combined, 3-part ratio that shows the relative portions of all the animals together

Notice that sheep appear in both ratios, so we can use sheep as the link between the two

C:S = 2:3 and S:G = 6:7

We can multiply both sides of the C:S ratio by 2, so that both ratios are comparing relative to 6 sheep

C:S = 4:6 and S:G = 6:7

These can now be joined together

C:S:G = 4:6:7

We can now use this to share the 85 animals in the **ratio 4:6:7**

There are 17 parts in total ($4 + 6 + 7 = 17$)

Each part is worth 5 animals ($85 \div 17 = 5$)

There are 20 cows (4×5), 30 sheep (6×5), and 35 goats (7×5)

Working with Proportion:

A way of talking about how two variables are related to each other



Solved Example:

The ingredients for 12 cookies are as follows:

90 g butter
130 g chocolate
180 g flour
150 g sugar
1 egg

(a) If Tanya makes 50 cookies, how much sugar does she need?

12 cookies require 150 g of sugar

Find how much sugar is needed to make 1 cookie (by dividing 150 by 12)

$$150 \div 12 = 12.5 \text{ grams of sugar for 1 cookie}$$

Find how much sugar is needed for 50 cookies (by multiplying 12.5 by 50)

$$12.5 \times 50 = 625$$

625 grams of sugar

(b) If Giles buys a 500 gram bag of flour to make cookies, how many whole cookies can he make?

Method 1

12 cookies require 180 g of flour

Find out how much flour is in 1 cookie (by dividing 180 by 12)

$$180 \div 12 = 15 \text{ grams of flour for 1 cookie}$$

Find how many lots of 15 grams are in 500 grams of flour (by dividing 500 by 15)

$$500 \div 15 = 33.33... \text{ cookies}$$

Write down how many whole cookies can be made

There is not enough for 34 whole cookies

33 cookies

Topic: Rates (1.12)

Major Terms and associated formulas:

Compound Measure:

Compound measure is something that is calculated by using more than one measurement.

Example:



Speed – how much the distance changes for each unit of time

Density – how heavy something is for each unit of its area or volume

Pressure – how much force is applied to an object for each unit of its area

Flow rate – how much the volume changes for each unit of time

Population density – how many people there are for each unit of area

Formulas: *Required formulas according to the question will be given.

Speed = Distance / Time

Density = Mass / Volume

Pressure = Force / Area

Understanding Time Units

(Time is one of the simplest concepts in the igcse mathematics syllabus which can give you a lot of problems if you have a weak foundation of the basic concept.)

We know that an hour hand takes 12 hours to go round the clock. It goes round the clock twice in a day so, 1 day = 24 hours. The minute hand takes 60 minutes to complete one round on the clock. So, 1 hour = 60 minutes.

Again, we know that there are 12 months in a year. The months of January, March, May, July, August, October and December have 31 days. The months of April, June, September and November have 30 days. The month of February has 28 or 29 days. In a week there are 7 days. 1 day is equal to 24 hours.

The standard units of time are:

1 hour = 60 minutes

1 minute = 60 seconds

1 hour = 60 minutes = 3600 seconds (60×60)

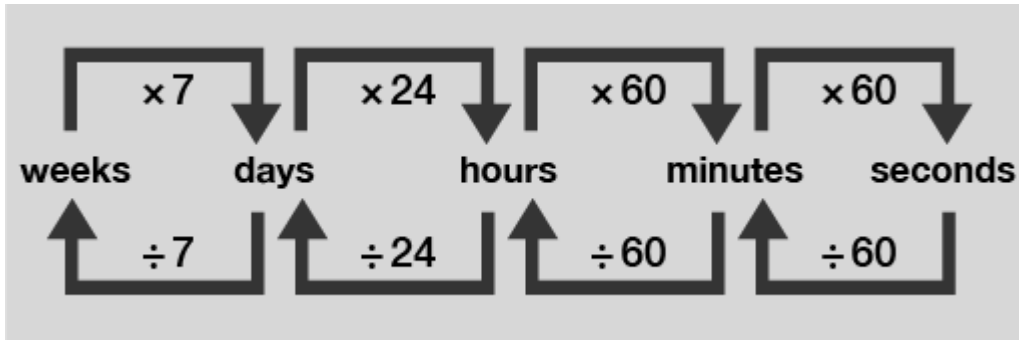
1 day = 24 hours

1 week = 7 days

1 year = 365 days

This simple but concise graph covers all the conversions you will need to know to attempt any questions about time.

Difference between 12 and 24 hour clock



1. 12-Hour Clock:

This clock divides the day into two halves: AM and PM.

1:00 AM is 1 hour after midnight

1:00 PM is 1 hour after noon

2. 24-Hour Clock:

This clock counts all the hours of the day from 0 to 23.

It starts at midnight as 00:00 and goes up to 23:59, just before the next midnight.

1:00 AM is written as 01:00.

1:00 PM is written as 13:00.

(Imagine you have a pizza that you cut into two halves. The first half is from midnight to noon (12 slices), and the second half is from noon to midnight (another 12 slices). In a 12-hour clock, you use the same numbers (1 to 12) twice. In a 24-hour clock, you keep going up to 23.)

Question: If it's 3:00 PM on a 12-hour clock, what would it be on a 24-hour clock?

Answer: It would be 15:00 on a 24-hour clock because you add 12 to the PM time ($3 + 12 = 15$).

The difference between different time zones

A time zone is a part of the world where everyone sets their clocks to the same time.

Question: If it's 3 PM in Tokyo (which is in a time zone that is 9 hours ahead of London's time zone), what time is it in London ?

Since Tokyo is ahead, you'll need to subtract 9 hours from 3 PM!

3 PM in Tokyo minus 9 hours = 6 AM in London.

So, when it's 3 PM in Tokyo, it's 6 AM in London.



(Just make sure you read the question carefully so you do not make any silly mistakes, leaving that the question is simple addition and subtraction.)

Basic Time Calculation:

Adding Time:

Question: If you start watching a movie at 2:30 PM and it lasts for 2 hours and 15 minutes, what time will it be when the movie ends?

Answer: 4:45 PM

Subtracting Time:

Question: A train departs at 11:20 AM and arrives at its destination at 1:10 PM. How long is the journey?

Answer: 1 hour and 50 minutes

Converting Time:

Question: Convert 150 minutes into hours and minutes.

Answer: 2 hours and 30 minutes

(The questions are a mix of these three conversions.)

Question: If a train departs at 10:15 AM and arrives at 1:45 PM, how long is the journey?

From 10:15 AM to 1:15 PM is 3 hours, then from 1:15 PM to 1:45 PM is 30 minutes.

Total journey time = 3 hours 30 minutes.

(Always remember to note AM/PM in questions. Double-check the time zone differences before calculating. Be aware of Daylight Saving Time changes in your calculations.)

Money

Money refers to the day-to-day currency that we use to buy items and for trading. In the IGCSE Mathematics, you would be expected to be familiar with basic arithmetic money calculations and conversions from one type of currency to another.

1.1 Calculations with Money:

For the 0580 Extended Course, you should be familiar with the following arithmetic operations:

1. Addition = +
2. Subtraction = -



3. Multiplication = *

4. Division = /

Let's take a question for an example.

Ali bought 10 candies for 12 dollars. Ahmed bought 15 candies for 18 dollars. They both paid collectively with a 50 dollar note. Calculate the total money paid for the bill and the change they will receive.

Reading the question more carefully makes us realise that we have to find two things:

1. Total money paid by them :

We might have to add the money paid by both $M_{total} = M_{Ali} + M_{Ahmed}$ (1)

2. Change they will receive which is the amount of money they will receive back so can be calculated as:

$$C = 50 - M_{total} \text{ (2)}$$

So now let's find out the individual answers. We know that the item bought by Ali costed 12 dollars and the item bought by Ahmed costed 18 dollars so we can say:

$$M_{total} = M_{Ali} + M_{Ahmed} \text{ (3)}$$

$$1 \ M_{total} = 12 + 18 \text{ (4)}$$

$$M_{total} = 30 \text{ (5)}$$

So we can say their bill was 30 dollars.

Now for the change, we can calculate it as:

$$C = 50 - M_{total} \text{ (6)}$$

$$C = 50 - 30 \text{ (7)}$$

$$C = 20 \text{ (8)}$$

So after paying with a 50 dollar bill, the change they will receive back would be 20 dollars.

1.2 Conversions between currencies

This part of the syllabus aims to assess your ability to convert from Currency A to currency B by using a multiplying factor, which will always be given in the question.

let's consider the following question to understand the expected concept:

1 dollar = 0.89 euros.

Ali spent 50 dollars on his trip to Paris. Ahmed spent 70 euros on his trip to Spain. Calculate the number of euros spent by Ali and the number of dollars spent by Ahmed.

Again, let's work part by part.

For the first, we've been asked to calculate and convert the dollars spent by Ali to euros so using the conversation rate we can say



(D stands for dollars and E stands for euros):

$$1D = 0.89E \text{ (9)}$$

$$50D = 0.89 * 50E \text{ (10)}$$

$$50D = 44.5E \text{ (11)}$$

Therefore, we can say that Ali spent 44.5 euros on his trip to Paris.

Now, for the next part, we have to do the opposite:

converting euros to dollars so we can say:

$$1D = 0.89E \text{ (12)}$$

$$1E = 1.089 D \text{ (13)}$$

$$70E = 1.089 * 70E \text{ (14)}$$

$$70E = 78.7E \text{ (15)}$$

Thus, the answer can be written as 78.7 euros, correct to three significant figures.

2.2 Exponential Growth and Decay Exponential growth

Refers to when a population of a sample grows very fast:

for example every hour it doubles.

Exponential decay refers to when a sample population decreases very rapidly:

for example every hour it halves. In either case, there's always a specific

multiplying factor that's used to find the value at the next hour. For exponential growth the factor would be greater than 1 and for decay, it would be less than 1. In either scenario, it would be positive (greater than 0).

2.1 Population Change

Let's consider population growth as an example. Assume that there's a population of bacteria growing in a laboratory. Every hour, the population doubles. Initially, the population was 2000. Find the population at the 5th hour. For question like this you have to form an exponential equation, because even though you can do it manually to get 4000, 8000, 16000, 32000 and so on, but sometimes the time can be very big so you need a formula to simplify things for you.

The general formula for such a case can be written as: $P = P_0(f)^n$ (16)

Here P represents the final population size,

P_0 represents the initial population size,

f is the factor by which it's growing or decaying and

finally n is the total time after which it's being observed.



Since it's doubling for our question, we can say that r is 2, the initial population P_0 would be 2000 and n would be 5 hours, therefore we can write it as: $P = 2000(2)^n$ (17) $P = 2000(2)^5$ (18) $P = 64000$ (19)

This makes it easier, right? Similarly if the question changed to exponential decay and instead of doubling, the population was halving the general formula could've been written as:

$$P = 2000\left(\frac{1}{2}\right)^n \quad (20)$$

Here, it would half every time. If they said it was decrease by a factor of 3 then it would've been:

$$P = 2000\left(\frac{1}{3}\right)^n \quad (21)$$

This same concept it also utilised in the compound interest formula: $C = P(1 \pm \frac{r}{100})^n$ (22)

3.3 Surds

Surds are values in square roots that can't be further simplified into a easier form.

Let's say we have square root of 40. If you put that in a calculator, it will, instead of outputting $\sqrt{40}$ give you the answer $2\sqrt{10}$. Why? .

The second form is the simplest form of $\sqrt{40}$. In this section we'll learn how to solve and simplify surds.

3.1 Simplifying Expressions involving surds

Let's say we have a surd $\sqrt{60}$ and we have to simplify it. For any question, the steps to be followed are:

1. For the number inside the square root, break it down into its factors as simply as possible so for 60 we can write it sequentially in fragments as:

$$(a) 2 \times 30$$

$$(b) 2 \times 2 \times 15$$

$$(c) 2 \times 2 \times 3 \times 5$$

Those numbers that appear in pairs, write them in exponent notation: so in the case of 60 as the number '2' appears two times, we can write the factors as $2^2 \times 3 \times 5$. The number that appeared in pairs and were subsequently put into the exponent form, bring the single number out of the root so we write it as:

$$2 \sqrt{3 \times 5} \quad (23) \quad 2 \sqrt{15} \quad (24)$$

Thus, this will be your final answer.



The property being used in these steps is as follows:

$$\sqrt{a} = \sqrt{x * y} = \sqrt{x} * \sqrt{y} = x \sqrt{y} \quad (25)$$

Here a is basically the initial number, x and y are your factors and they can be split into separate square roots. Assuming that x appeared in a pair, its squared root can be cut and will be only left as a single number outside the square root. The remaining number y will remain inside in its initial form.

3.2 Rationalisation of Surds

Rationalisation refers to the process of changing a fraction that has surd in its denominators into a denominator with a whole number. The procedure for it, in general form, is as follows:

4 1. Look at the denominator and find its real conjugate:

conjugate refers to the alternate form of the denominator so for single surds like $\sqrt{5}$ its the same $\sqrt{5}$ but for denominators like $1 + \sqrt{5}$, always change the sign in between to find the conjugate as $1 - \sqrt{5}$ so as a rule of thumb:

for compound conjugates, if there's a plus sign, change it to a minus and if there's a minus, change it to a plus sign.

2. Multiply the surd fraction's numerator and denominator given in the question with the conjugate your just found.

3. Use basic algebraic simplifications or sometimes surd simplification to give the final answer in the simplest terms possible. Let's consider an example as follows:

Simplify $2 \frac{1+\sqrt{5}}{1-\sqrt{5}}$

Now we can simplify it as:

$$2 \frac{1 + \sqrt{5}}{1 - \sqrt{5}} * \frac{(1 + \sqrt{5})}{(1 + \sqrt{5})} \frac{(1 - \sqrt{5})}{(1 - \sqrt{5})} \quad (26) \quad 2 \frac{(1 + \sqrt{5})(1 - \sqrt{5})}{(1 - \sqrt{5})(1 + \sqrt{5})} \quad (27)$$

$$2 \frac{(1 - \sqrt{5})(1 + \sqrt{5})}{(1 - \sqrt{5})(1 + \sqrt{5})} \quad (28) \quad 2 \frac{(1 - \sqrt{5})(1 + \sqrt{5})}{(1 - \sqrt{5})(1 + \sqrt{5})} \quad (29) \quad 2 \frac{(1 - \sqrt{5})(1 + \sqrt{5})}{(1 - \sqrt{5})(1 + \sqrt{5})} \quad (30) \quad 2 \frac{(1 - \sqrt{5})(1 + \sqrt{5})}{(1 - \sqrt{5})(1 + \sqrt{5})} \quad (31) \quad 1 - \sqrt{5} - 2 \quad (32)$$

Multiplying out by -1 gives us the final answer as: $\sqrt{5} - 1 \quad 2 \quad (33)$



UNSOLVED PAST PAPER QUESTIONS:

2

- 1 The temperature at midnight is -4°C .
The temperature at noon is 25°C .

Work out the difference between these two temperatures.

..... $^{\circ}\text{C}$ [1]

- 2 A gardener charges \$6.55 for each hour he works plus a fixed charge of \$15.50 .

Calculate the total amount he charges when he works for 4 hours.

-
- 4 Jonah has \$750.
He spends $\frac{1}{4}$ of this money on travel and some of this money on food.
He now has \$437.50 .

Work out the fraction of the \$750 he spends on food.

..... [3]

- 5 The table shows part of a tram timetable.

Newpoint	Westhill
10 30	11 17
12 18	
13 30	14 17

All the trams take the same number of minutes to complete the journey from Newpoint to Westhill.

Complete the table.

[2]

**ANSWER KEY:**

2

- 1 The temperature at midnight is -4°C .
The temperature at noon is 25°C .

Work out the difference between these two temperatures.

$$= 25 - (-4)$$

$$= 25 + 4$$

$$= 29$$

..... 29 $^{\circ}\text{C}$ [1]

- 2 A gardener charges \$6.55 for each hour he works plus a fixed charge of \$15.50.

Calculate the total amount he charges when he works for 4 hours.

$$(6.55 \times 4) + 15.50$$

$$26.2 + 15.5$$

\$ 41.7 [2]

3

- 4 Jonah has \$750.

He spends $\frac{1}{4}$ of this money on travel and some of this money on food.

He now has \$437.50.

Work out the fraction of the \$750 he spends on food.

$$\text{\$ on Travel} = \frac{1}{4} \times 750 = 187.5$$

$$\text{\$ on food} = (750 - 437.5) - 187.5$$

$$= 312.5 - 187.5$$

$$= 125$$

$$\frac{125}{750} = \frac{1}{6}$$

..... $\frac{1}{6}$ [3]

- 5 The table shows part of a tram timetable.

Newpoint	Westhill
10 30	11 17
12 18	<u>13 05</u>
13 30	14 17

All the trams take the same number of minutes to complete the journey from Newpoint to Westhill.

Complete the table.

$$11 : 17$$

$$= 10 : 30$$

$$\begin{array}{r|l} 10 : 30 & 12 : 18 \\ 10 : 30 & + 00 : 47 \\ \hline 0 : 47 & 12 : 65 \\ & + 1 : 00 \\ \hline & 13 : 05 \end{array}$$

..... 13 05 [1]

- 6 Write 0.04628 correct to 2 significant figures.



Unit 2: Algebra and Graphs

2 Algebra and graphs

E2.1 Introduction to algebra

Notes and examples

- 1 Know that letters can be used to represent generalised numbers.
- 2 Substitute numbers into expressions and formulas.

E2.2 Algebraic manipulation

Notes and examples

- 1 Simplify expressions by collecting like terms.
- 2 Expand products of algebraic expressions.
- 3 Factorise by extracting common factors.
- 4 Factorise expressions of the form:
 - $ax + bx + kay + kby$
 - $a^2x^2 - b^2y^2$
 - $a^2 + 2ab + b^2$
 - $ax^2 + bx + c$
 - $ax^3 + bx^2 + cx$.
- 5 Complete the square for expressions in the form $ax^2 + bx + c$.

Simplify means give the answer in its simplest form,
e.g. $2a^2 + 3ab - 1 + 5a^2 - 9ab + 4 = 7a^2 - 6ab + 3$.
e.g. expand $3x(2x - 4y)$, $(3x + y)(x - 4y)$.
Includes products of more than two brackets,
e.g. expand $(x - 2)(x + 3)(2x + 1)$.
Factorise means factorise fully,
e.g. $9x^2 + 15xy = 3x(3x + 5y)$.

E2.3 Algebraic fractions

Notes and examples

- 1 Manipulate algebraic fractions.

Examples include:

- $\frac{x}{3} + \frac{x-4}{2}$
- $\frac{2x}{3} - \frac{3(x-5)}{2}$
- $\frac{3a}{4} \times \frac{9a}{10}$
- $\frac{3a}{4} \div \frac{9a}{10}$
- $\frac{1}{x-2} + \frac{x+1}{x-3}$.

- 2 Factorise and simplify rational expressions.

e.g. $\frac{x^2 - 2x}{x^2 - 5x + 6}$.



2 Algebra and graphs (continued)

E2.4 Indices II

- 1 Understand and use indices (positive, zero, negative and fractional).
- 2 Understand and use the rules of indices.

Notes and examples

e.g. solve:

- $32^x = 2$
- $5^{x+1} = 25^x$.

e.g. simplify:

- $3x^{-4} \times \frac{2}{3}x^{\frac{1}{2}}$
- $\frac{2}{5}x^{\frac{1}{2}} \div 2x^{-2}$
- $\left(\frac{2x^4}{3}\right)^3$.

Knowledge of logarithms is **not** required.

E2.5 Equations

- 1 Construct expressions, equations and formulas.
- 2 Solve linear equations in one unknown.
- 3 Solve fractional equations with numerical and linear algebraic denominators.
- 4 Solve simultaneous linear equations in two unknowns.
- 5 Solve simultaneous equations, involving one linear and one non-linear.
- 6 Solve quadratic equations by factorisation, completing the square and by use of the quadratic formula.
- 7 Change the subject of formulas.

Notes and examples

e.g. write an expression for the product of two consecutive even numbers.

Includes constructing simultaneous equations.

Examples include:

- $3x + 4 = 10$
- $5 - 2x = 3(x + 7)$.

Examples include:

- $\frac{x}{2x+1} = 4$
- $\frac{2}{x+2} + \frac{3}{2x-1} = 1$
- $\frac{x}{x+2} = \frac{3}{x-6}$.

With powers no higher than two.

Includes writing a quadratic expression in completed square form.

Candidates may be expected to give solutions in surd form.

The quadratic formula is given in the List of formulas.

e.g. change the subject of a formula where:

- the subject appears twice
- there is a power or root of the subject.



2 Algebra and graphs (continued)

E2.6 Inequalities

Notes and examples

- 1 Represent and interpret inequalities, including on a number line.

When representing and interpreting inequalities on a number line:

- open circles should be used to represent strict inequalities ($<$, $>$)
- closed circles should be used to represent inclusive inequalities ($\leq$, $\geq$).

e.g. $-3 \leq x < 1$



- 2 Construct, solve and interpret linear inequalities.

Examples include:

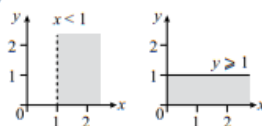
- $3x < 2x + 4$
- $-3 \leq 3x - 2 < 7$.

- 3 Represent and interpret linear inequalities in two variables graphically.

The following conventions should be used:

- broken lines should be used to represent strict inequalities ($<$, $>$)
- solid lines should be used to represent inclusive inequalities ($\leq$, $\geq$)
- shading should be used to represent unwanted regions (unless otherwise directed in the question).

e.g.



- 4 List inequalities that define a given region.

Linear programming problems are **not** included.

E2.7 Sequences

Notes and examples

- 1 Continue a given number sequence or pattern.
- 2 Recognise patterns in sequences, including the term-to-term rule, and relationships between different sequences.
- 3 Find and use the n th term of sequences.

Subscript notation may be used, e.g. T_n is the n th term of sequence T .

Includes linear, quadratic, cubic and exponential sequences and simple combinations of these.



2 Algebra and graphs (continued)

E2.8 Proportion

Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities.

Notes and examples

Includes linear, square, square root, cube and cube root proportion.
Knowledge of proportional symbol ($\propto$) is required.

E2.9 Graphs in practical situations

- 1 Use and interpret graphs in practical situations including travel graphs and conversion graphs.
- 2 Draw graphs from given data.
- 3 Apply the idea of rate of change to simple kinematics involving distance–time and speed–time graphs, acceleration and deceleration.
- 4 Calculate distance travelled as area under a speed–time graph.

Notes and examples

Includes estimation and interpretation of the gradient of a tangent at a point.

Areas will involve linear sections of the graph only.

E2.10 Graphs of functions

- 1 Construct tables of values, and draw, recognise and interpret graphs for functions of the following forms:
 - ax^n (includes sums of no more than three of these)
 - $ab^x + c$
 where $n = -2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2, 3$; a and c are rational numbers; and b is a positive integer.
- 2 Solve associated equations graphically, including finding and interpreting roots by graphical methods.
- 3 Draw and interpret graphs representing exponential growth and decay problems.

Notes and examples

Examples include:

- $y = x^3 + x - 4$
- $y = 2x + \frac{3}{x^2}$
- $y = \frac{1}{4} \times 2^x$

e.g. finding the intersection of a line and a curve.



E2.12 Differentiation

Notes and examples

- 1 Estimate gradients of curves by drawing tangents.
- 2 Use the derivatives of functions of the form ax^n , where a is a rational constant and n is a positive integer or zero, and simple sums of not more than three of these.
- 3 Apply differentiation to gradients and stationary points (turning points).
- 4 Discriminate between maxima and minima by any method.

$\frac{dy}{dx}$ notation will be expected.

Maximum and minimum points may be identified by:

- an accurate sketch
- use of the second differential
- inspecting the gradient either side of a turning point.

Candidates are **not** expected to identify points of inflection.

2 Algebra and graphs (continued)

E2.13 Functions

Notes and examples

- 1 Understand functions, domain and range and use function notation.
- 2 Understand and find inverse functions $f^{-1}(x)$.
- 3 Form composite functions as defined by $gf(x) = g(f(x))$.

Examples include:

- $f(x) = 3x - 5$
- $g(x) = \frac{3(x+4)}{5}$
- $h(x) = 2x^2 + 3$.

e.g. $f(x) = \frac{3}{x+2}$ and $g(x) = (3x+5)^2$. Find $fg(x)$.
Give your answer as a fraction in its simplest form.

Candidates are **not** expected to find the domains and ranges of composite functions.

This topic may include mapping diagrams.

2.1 Algebraic Notation

What is algebraic notation?

When writing expressions in algebra (as opposed to sums in numbers) there are conventions and symbols that are used that take on a particular meaning

This is what we mean by algebraic notation

In number work, for adding and subtracting, we use + and –

In algebra, we still do!



Examples:

$$a + b$$

$$c + d - e$$

However for multiplication, no symbol is used, and for division, fractions are used

Examples:

$$ab \text{ (means } a \times b \text{)}$$

$$\frac{a}{b} \text{ (means } a \div b \text{)}$$

$$3ab \text{ (means } 3 \times a \times b \text{)}$$

We can have combinations of these of course

Powers (indices) and roots are used the same way as with numbers

Examples:

$$a^2 \text{ means } a \times a$$

$$4a^2 \text{ means } 4 \times a^2$$

With the order of operations, a^2 will happen before multiplying by 4

Brackets also work in the same way as with numbers

How will I need to use algebraic notation?

Algebraic expressions are used in many parts of the course

You need to be able to understand their meaning and work with them

e.g. for rearranging a formula

You may be given a situation in words that you then have to write in algebra

Which could lead to an equation you may then have to solve

Worked example

At the start of a competition, Raheem has p conkers and Howard has $2q$ conkers.

At the end of the competition Raheem still has the same number of conkers he started with but Howard won 6 and lost none.

(a)

Write down an expression for the number of conkers that Howard has at the end of the competition.

$$\text{Howard has } 2q + 6$$

At the end of the competition, Raheem and Howard have a total of 40 conkers.

(b)

Write an equation in terms of p and q that shows the number of conkers Raheem and Howard have in total at the end of the competition.

$$p + 2q + 6 = 40$$

This can be simplified to $p + 2q = 34$

What is a term?

A term is either...

...a letter (variable) on its own, e.g. x

..a number on its own, e.g. 20



...a number multiplied by a letter, e.g. $5x$

The number in front of a letter is called a coefficient

The coefficient of x in the term $6x$ is 6

The coefficient of y in the term $-5y$ is -5

Terms that are just numbers (with no letters) are called constants

Terms can include powers and more than one letter

E.g. $6xy$, $4x^2$, ab^3c , ...

What is a factor?

A factor is any number or letter that divides a term exactly (with no remainder)

E.g. all the factors of $4xy$ are 1, 2, 4, x , $2x$, $4x$, y , $2y$, $4y$, xy , $2xy$ and $4xy$

A term can be separated into factors that multiply together to give that term

E.g. two factors of $5x$ are 5 and x

To factorise means to write something as a multiplication of factors

A common factor is one that divides both terms

The highest (or greatest) common factor is $2x$

E.g. the common factors of $6xy$ and $4x$ are 2, x and $2x$

What is an expression?

An expression is an algebraic statement that does not have an equals sign

There is nothing to solve

An expression is made by adding, subtracting, multiplying or dividing terms

E.g. $2x + 5y$, $b^2 - 2cd$, , ...

A single term can be an expression

Expressions can be simplified (made easier)

E.g. $x + x + x$ simplifies to $3x$

What is an equation?

An equation is an algebraic statement with an equals sign between a left-hand side and a right-hand side

Both sides are equal in value

E.g. if $2x$ has the same value as 10, then $2x = 10$

An equation can be solved by finding the missing values of the letters that make the left-hand side equal to the right-hand side

$x = 5$ is called the solution

E.g. the equation $2x = 10$ is solved by $x = 5$

What is a formula?

A formula is a worded rule, definition or relationship between different quantities, written in shorthand using letters

The formula is $w = mg$

E.g. weight, w , is mass, m , multiplied by gravitational acceleration, g

It is common to substitute numbers into a formula, but a formula on its own cannot be solved

To turn a formula into an equation, more information is needed



E.g. In the formula $w = mg$, if $w = 50$ and $m = 5$ then the equation $50 = 5g$ can be formed

2.2 Substitution

What is substitution?

Substitution is where we replace letters in a formula with their values

This allows you to find one other value that is in the formula

How do we substitute numbers into a formula?

Write down the formula, if not clearly stated in question

Substitute the numbers given, using brackets around negative numbers, (-3) , (-5) etc

Simplify any calculations if you can

Rearrange the formula if necessary (it is usually easier to substitute first)

Work out the calculation (use a calculator if allowed)

Are there any common formulae to be aware of?

The formula for the equation of a straight line is often used

$y = mx + c$



WORKED EXAMPLE:

- (a) Find the value of the expression $2x(x + 3y)$ when $x = 2$ and $y = -4$.

Substitute the numbers given.

Use brackets $()$ around negative numbers that you have substituted so that you don't forget about them.

It is a good idea to show every step of working to make sure that you are following the order of operations correctly.

$$\begin{aligned} & 2 \times 2 \times (2 + 3 \times (-4)) \\ &= 2 \times 2 \times (2 - 12) \\ &= 2 \times 2 \times (-10) \\ &= 4 \times -10 \end{aligned}$$

- 40

- (b) The formula $P = 2l + 2w$ is used to find the perimeter, P , of a rectangle of length l and width w .
Given that the rectangle has a perimeter of 20 cm and a width of 4 cm, find its length.

Substitute the values you are given into the formula.

$$20 = 2 \times l + 2 \times 4$$

Simplify.

$$20 = 2l + 8$$

Subtract 8 from both sides.

$$12 = 2l$$

Divide both sides by 2.

$l = 6$ cm

Collecting Like Terms

How do we collect like terms?

TERMS are separated by + or -

The sign belongs to the coefficient of the term after the symbol

If there is no symbol in front of the first term then this is a positive term

$2x - 3y$ means +2 x's and -3 y's

"LIKE" terms must have exactly the same LETTERS AND POWERS (the COEFFICIENT can be different)

Examples of like terms:

$2x$ and $3x$

$2x^2$ and $3x^2$

$2xy$ and $3xy$

$4(x + y)$ and $5(x + y)$

Examples that are NOT like terms



$2x$ and $3y$ (different letters)

$2x^2$ and $3x^4$ (different powers)

$2xy$ and $3xyz$ (different letters)

$4(x + y)$ and $5(x + y)^2$ (different powers)

Remember multiplication can be done in any order

xy and yx are like terms

Add the COEFFICIENTS of like terms

If the answer is a positive answer then put "+" in front if there are other terms before it

$$x - 2y + 5y = x + 3y$$

If the answer is a negative number then put "-" in front

$$x - 5y + 2y = x - 3y$$

Worked example

Simplify $x^2 - 3xy + 2x^2 + 4x - 2xy - x + 7$.

Reorder the terms so that the x^2 's are together, as are the xy 's, x 's and the constants.

Make sure that you keep the same sign in front of them when you reorder them.

$$x^2 + 2x^2 - 3xy - 2xy + 4x - x + 7$$

Add the coefficients of the 'like' terms.

$$3x^2 - 5xy + 3x + 7$$

2.6 Inequalities Notes

1. Understanding Inequalities

An inequality compares two values and shows the relationship between them.

Common symbols:

$<$ less than

$>$ greater than

$\leq$ less than or equal to

$\geq$ greater than or equal to

2. Types of Inequalities

Linear inequalities

Quadratic inequalities

3. Solving Linear Inequalities

Step 1: Isolate the variable (similar to solving equations).

Step 2: If you multiply or divide by a negative number, reverse the inequality sign.

4. Solving Quadratic Inequalities

Step 1: Factor the quadratic expression (if possible).



Step 2: Find the roots (where the expression equals zero).

Step 3: Test intervals to determine where the inequality holds.

5. Representing Solutions on a Number Line

Open Circle: Use for $<$ and $>$ (not including the endpoint).

Closed Circle: Use for $\leq$ and $\geq$ (including the endpoint).

Inequalities - drawing / using pictures		
Inequality	Picture	Extra Q
$-2 < x < 4$		list integers x could be $-1, 0, 1, 2, 3$
$-3 < x \leq 5$		$-2, -1, 0, 1, 2, 3, 4, 5$
$-4 \leq x \leq 3$		$-4, -3, -2, -1, 0, 1, 2, 3$
$-2 \leq x < 5$		$-2, -1, 0, 1, 2, 3, 4$
$x \leq -1$ or $x > 2$		

6. Graphical Representation of Quadratic Inequalities

Use a number line or a coordinate plane to show the regions where the inequality is satisfied. Then plot the roots and use open or closed circles depending on the inequality sign.

Graphing Linear Inequalities

STEP ONE

Build the line using slope and y-intercept

$$m = 3 \quad b = -2$$

STEP TWO

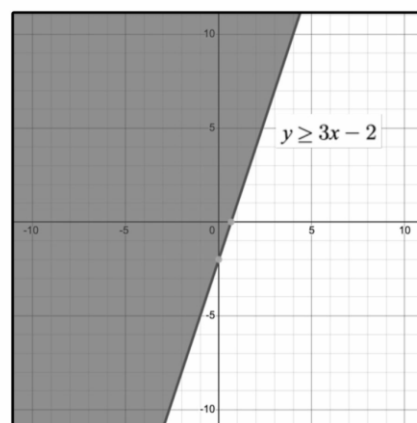
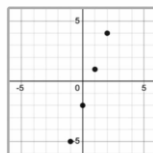
Graph the line (solid or dotted)

STEP THREE

Fill in the shaded region and identify the solution set.

$\geq$ or $\leq$: solid line
 $>$ or $<$: dotted line

$\geq$ or $>$: shade above
 $\leq$ or $<$: shade below



(Always check your solutions by substituting back into the original inequality.)



When graphing, clearly label your number lines and intervals. Practice with various types of inequalities to solidify your understanding.)

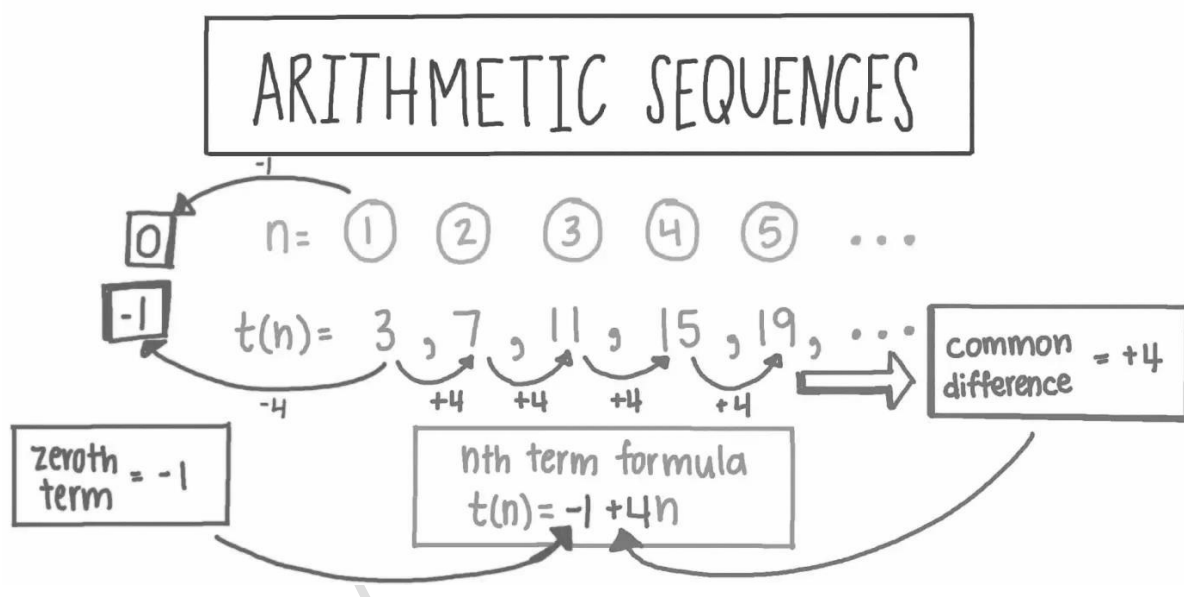
2.7 Sequences Notes

1. Understanding Sequences

A sequence is an ordered list of numbers, called terms. A rule or formula can describe sequences.

2. Simple Sequences

A simple sequence can be generated by counting or adding a constant.



3. Linear Sequences

A linear sequence has a constant difference between consecutive terms.

1) 6, 10, 14, 18, 22 The sequence increases by 4, so the nth term starts with $4n$

Now compare the sequence to the 4 times table

6, 10, 14, 18, 22 Each term is 2 bigger than the 4 times table

↑ +2 ↑ +2 ↑ +2 ↑ +2 ↑ +2

4, 8, 12, 16, 20

So the nth term is $4n + 2$



4. Finding Terms in Linear Sequences

To find a specific term, substitute n into the general term formula.

Example 9.

Given the following sequence, what is the 28th term?

18, 21, 24, 27, 30

Solution.

- 1. Find the common difference.**
- 2. Generate a formula that uses the variable n .**
- 3. Use the formula to find the n th term.**

The common difference is 3.

The n_0 term is 18.

The formula for generating terms is

$$a_n = 18 + (n - 1) \cdot 3$$

$$\begin{aligned} a_{28} &= 18 + (28 - 1) \cdot 3 \\ &= 18 + 27 \cdot 3 \\ &= 99 \end{aligned}$$

5. Quadratic Sequences

A quadratic sequence has a second difference that is constant. The difference between the terms changes linearly.

General form

a, b, c (constants)

Finding the formula:

Identify the first differences (subtract each term from the next).

Identify the second differences (subtract the first differences).

Use the values to form a system of equations to find a, b , and c .



Finding an nth term rule for a quadratic sequence

For a quadratic sequence $an^2 + bn + c$

the 2nd difference is always $2a$

The first 1st difference is always $3a + b$

the 1st term is always $a + b + c$

Eg find the nth term rule of the sequence 8, 14, 22, 32, ...

$a + b + c = 8$	14	22	32	sequence
-----------------	----	----	----	----------

$\Rightarrow c = 5$

$3a + b = 6$	8	10	1 st differences
--------------	---	----	-----------------------------

$\Rightarrow b = 3$

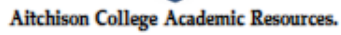
$2a = 2$	2	2 nd differences
----------	---	-----------------------------

$\Rightarrow a = 1$

so the nth term rule is $n^2 + 3n + 4$

(Identify the type of sequence before trying to find a general term. Always check your terms by substituting back into the formula. Practice different types of sequences to strengthen your understanding.)

The next page has a cheat sheet for you guys, showing the different sequences that might appear in the exam.



TYPES OF SEQUENCES

LINEAR

e.g. $2 \xrightarrow{+4} 6 \xrightarrow{+4} 10 \xrightarrow{+4} 14 \xrightarrow{+4} 18 \dots$

FIRST DIFFERENCES	CONSTANT
-------------------	----------

QUADRATIC

e.g.

5	9	15	23	33	...
↗		↗		↗	
+4		+6		+8	
↗			↗		
+2			+2		

SECOND DIFFERENCES CONSTANT

GEOMETRIC

e.g. $4 \quad 12 \quad 36 \quad 108 \quad 324 \quad \dots$
 $\quad \quad \quad \nearrow \quad \nearrow \quad \nearrow \quad \nearrow$
 $\quad \quad \quad \times 3 \quad \times 3 \quad \times 3 \quad \times 3$

CONSTANT	MULTIPLIER	(COMMON RATIO)
----------	------------	----------------

FIBONACCI

e.g.

The diagram illustrates the generation of the sequence 2, 4, 6, 10, 16, 26, 42, ... using a sliding window of size 2. Brackets connect pairs of numbers, and arrows with a plus sign indicate the addition of each pair to the next number in the sequence:

- 2 and 4 are added to get 6.
- 4 and 6 are added to get 10.
- 6 and 10 are added to get 16.
- 10 and 16 are added to get 26.
- 16 and 26 are added to get 42.

The sequence continues with an ellipsis (...).

ADD THE PREVIOUS TWO TERMS

OTHER

e.g.	n	1	2	3	4	
		1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	...

n^{th} TERM, $a_n = \frac{1}{n}$

SUCH SEQUENCES DON'T FALL INTO ANY CATEGORY BUT THE LINK BETWEEN n AND a_n IS FAIRLY EASY TO SPOT

n^{th} TERM, $a_n = \frac{1}{n}$

SUCH SEQUENCES DON'T FALL INTO ANY CATEGORY BUT THE LINK BETWEEN n AND a_n IS FAIRLY EASY TO SPOT



Interpreting Graphs

Interpreting graphs involves understanding what the shape of a curve represents in real life situations. We should also understand what slope means and how to interpret what a high or low slope value represents. An understanding of plotting points is also important. Graphs communicate important quantitative information in a visual format. Using this form of representation, students must be able to:

- Elicit key information
- Extract the basic information
- Draw conclusions
- Compare and contrast situations and
- Provide recommendations.

Solution of Equations

Accurately drawn graphs enables us to find approximate solutions to a wide range of equations, many of which are impossible to solve exactly 'conventional' methods.

- Draw the graph of a given function for given range.
- To find the solutions of equation rearrange the equation so that the function already plotted is on one side.
- Draw the graph for the other side of the equation.
- Now the values of the point of intersection are the solutions of the equation.

Distance-Time Graph

When a distance-time graph is drawn the gradient of a graph gives the speed of an object. (Because)

- O to A: constant speed
- A to B: speed goes down to zero
- B to C: at rest
- C to D: accelerates (speed is increasing)
- D to E: constant speed (not as fast as O to A)

Speed-Time Graph

When a speed-time graph is drawn the gradient of a graph gives the acceleration of an object and area under the graph tells us about the distance travelled by that object. (

Turning Points

A turning point (also known as stationary point) is a point on a graph where the gradient is zero. One method that can be used to find the turning points of a certain graph is completing the square method. When the quadratic function is written in the form the coordinates of the turning point on the graph are (a, b) . This method is used to find the turning points on a quadratic graph, but for other graphs (for example cubic graphs) this does not work. For such graphs we use gradient functions.

Gradient Functions



The gradient function is called the derivative function and, if, it is denoted by (the derivative of with respect to, or by, or using the notation.

The rule for finding the derivative function for integer powers of is:

(‘Multiply by power and reduce by one.)

The process of finding the derivative function is differentiation. (Constant term disappears). To find the gradient of a curve at a particular point, substitute the given x-value into the derivative function.

Turning points of the graphs (and quadratic graphs) can be found by setting the derivative function equal to zero and solving the resulting equation.

Second Derivative

There are two type of turning points in the graphs, maximum and minimum. Find the second derivative and substitute the coordinates of the turning point in. The *sign* of the second derivative determines whether the point is maximum or minimum.

Differentiation

→ Power Functions (Examples)

1. $y = 1x^3$

$$\frac{dy}{dx} = 3x^2$$

2. $y = 5x^4$

$$\frac{dy}{dx} = 20x^3$$

3. $y = \sqrt{x}$

$$\textcircled{1} y = ax^n$$

$$\frac{dy}{dx} = nax^{n-1}$$

$$\textcircled{2} y = c$$

$$\frac{dy}{dx} = 0$$



$$y = x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} x^{\frac{1}{2}-1}$$

$$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \quad \text{or} \quad \frac{1}{2\sqrt{x}}$$

$$4. \quad y = 3x^3 + 5x^2 - 7x + 1x^0$$

$$\frac{dy}{dx} = 9x^2 + 10x - 7 + 0$$

$$y = 3x^2 - 6x + 1, \text{ find the gradient of the curve at } x=3$$

$$\frac{dy}{dx} = 6x - 6$$

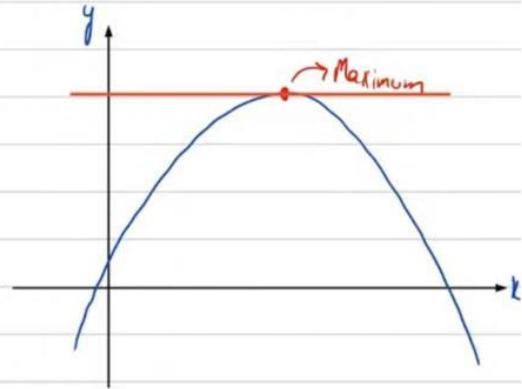
$$\text{Plug in } x=3$$

$$\frac{dy}{dx} = 6(3) - 6$$

$$\frac{dy}{dx} = 12$$



Maximum/Minimum/Stationary/Turning points



Note: Both, min & max points are also referred to as stationary or turning points.

Gradient of a horizontal line is always zero.

How to find turning/stationary point?

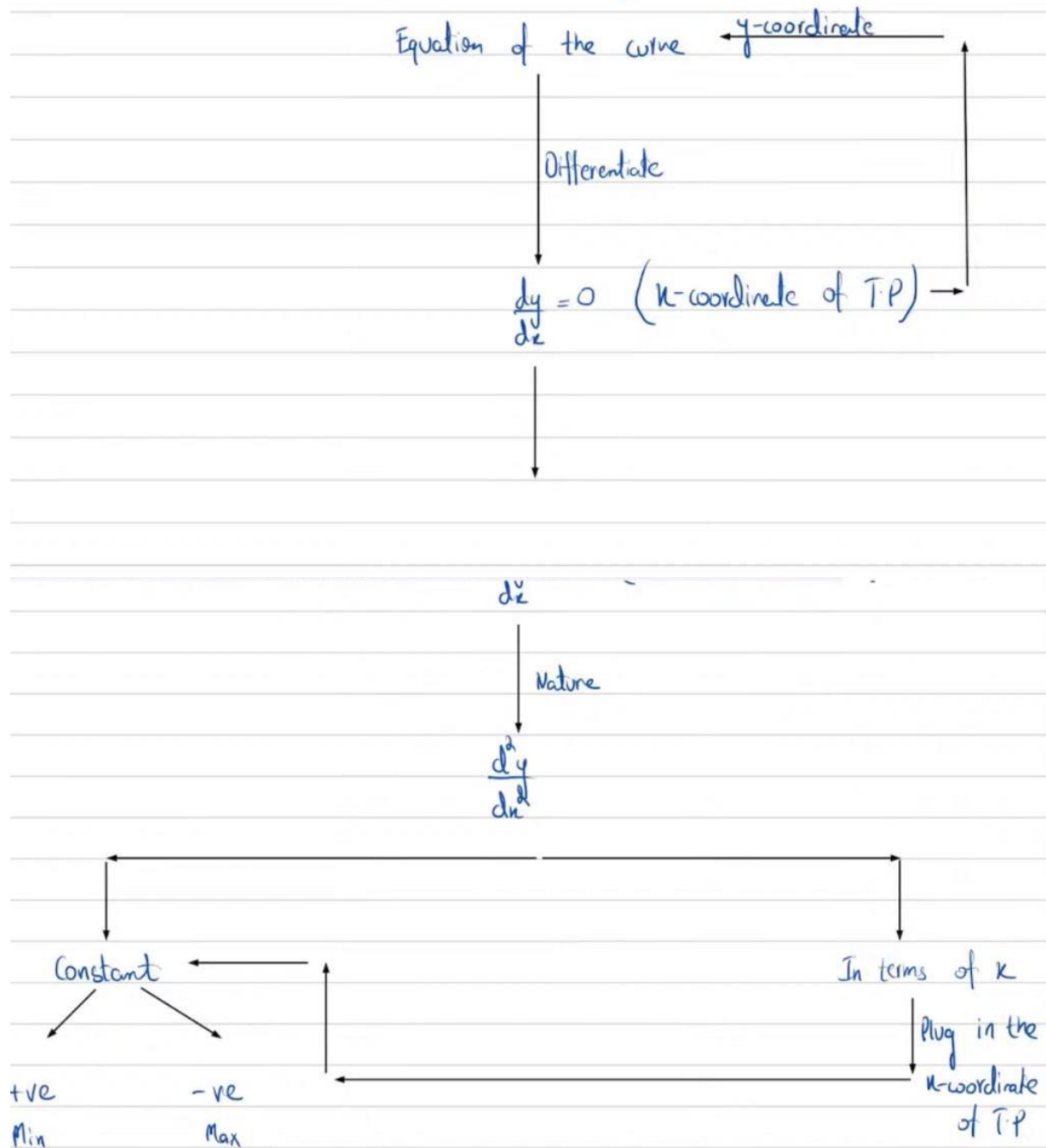
① Find $\frac{dy}{dx}$

② Equate $\frac{dy}{dx} = 0$ & solve for x

③ Plug in the value of x , obtained in step 2, to find the y -coordinate.



How to determine the nature of the turning point?





SOLVED EXAMPLE:

- (ii) Find the two stationary points on the graph of $y = x^4 - 4x^3$.

$$\begin{aligned}\frac{dy}{dx} &= 4x^3 - 12x^2 \\ 4x^3 - 12x^2 &= 0 \\ 4x^2(x - 3) &= 0 \\ 4x^2 = 0 \quad \text{or} \quad x - 3 &= 0 \\ x = 0 \quad \text{or} \quad x = 3 \\ y = 0^4 - 4(0)^3 &= 0 \\ y = 3^4 - 4(3)^3 &= -27 \\ &(\dots 0 \dots, \dots 0 \dots) \\ &(\dots 3 \dots, \dots -27 \dots) \\ &(\dots \frac{1}{2} \dots, \dots -2 \dots) \text{ and } (\dots -\frac{1}{2} \dots, \dots 4 \dots) [5]\end{aligned}$$

- (ii) Determine whether each of the stationary points is a maximum or a minimum.
Give reasons for your answers.

$$\begin{aligned}\frac{dy}{dx} &= 12x^2 - 3 \\ \frac{d^2y}{dx^2} &= 24x \\ \swarrow & \searrow \\ x = \frac{1}{2} & \quad x = -\frac{1}{2} \\ \text{+ve} & \quad \text{-ve} \\ \text{Min} & \quad \text{Max.}\end{aligned} \quad [3]$$



What is a function?

A function is a combination of one or more mathematical operations that takes a set of numbers and changes them into another set of numbers

It may be thought of as a mathematical “machine”

For example, if the function (rule) is “double the number and add 1”, the two mathematical operations are “multiply by 2 ($\times 2$)” and “add 1 ($+1$)”

Putting 3 in to the function would give $2 \times 3 + 1 = 7$

Putting -4 in would give $2 \times (-4) + 1 = -7$

Putting x in would give $2x+1$

The number being put into the function is often called the input

The number coming out of the function is often called the output

What does a function look like?

A function f can be written as $f(x) = \dots$ or $f : x \mapsto \dots$

These two different types of notation mean exactly the same thing

Other letters can be used. g , h and j are common but any letter can technically be used

Normally, a new letter will be used to define a new function in a question

For example, the function with the rule “triple the number and subtract 4” would be written

$$F_x = 3x - 4$$

In such cases, x would be the input and y would be the output

Sometimes functions don't have names like f and are just written as $y = \dots$

eg. $3x - 4$

How does a function work?

A function has an input x and output $f(x)$ or y

Whatever goes in the bracket (instead of x) with f , replaces the y on the other side

This is the input

If the input is known, the output can be calculated

If the output is known, an equation can be formed and solved to find the input

Composite Functions

What is a composite function?

A composite function is one function applied to the output of another function

Composite functions may also be referred to as compound functions

What do composite functions look like?

The notation you will see for a composite function is $fg(x)$

This can be written as $f(g(x))$ and means “ f applied to the output of $g(x)$ ”

i.e. $g(x)$ happens first

Always apply the function on the outside to the output of the function on the inside

$gf(x)$ means $g(f(x))$ and means “ g applied to the output of $f(x)$ ”

i.e. $f(x)$ happens first

How does a composite function work?



If you are putting a number into $fg(x)$

STEP 1

Put the number into $g(x)$

STEP 2

Put the output of $g(x)$ into $f(x)$

If you are using algebra, to find an expression for a composite function

STEP 1

For $fg(x)$ put $g(x)$ wherever you see x in $f(x)$

STEP 2

Simplify if necessary

An inverse function does the exact opposite of the function it came from

For example, if the function “doubles the number and adds 1” then its inverse is “subtract 1 and halve the result”

It is the inverse operations in the reverse order

How do I write inverse functions?

An inverse function f^{-1} can be written as $F^{-1}(x)$

How do I find an inverse function?

The easiest way to find an inverse function is to 'cheat' and swap the x and y variables

Note that this is a useful method but you **MUST** remember not to do this in any other circumstances in maths

STEP 1

Write the function in the form $y = \dots$

STEP 2

Swap the X s and Y s to get $x = \dots$

STEP 3

Rearrange the expression to make y the subject again

STEP 4

Write as $f^{-1}(x) = \dots$ (or $f^{-1} : x \mapsto \dots$)



UNSOLVED PAST PAPER QUESTIONS:

- (b) Expand the brackets and simplify.

$$4(3x - 2) - 3(x - 5)$$

..... [2]

- (c) Factorise completely.

$$12ab - 20a^2$$

- 3 Rearrange the formula to make w the subject.

$$5w - 3y + 7 = 0$$

-
- 6 Simplify.

$$\frac{2x^2 + x - 15}{ax + 3a - 2bx - 6b}$$

-
- (e) Solve the simultaneous equations.
You must show all your working.

$$\begin{aligned}x - 3y &= 13 \\ 2x^2 - 9y &= 116\end{aligned}$$



9 (a) Simplify.

(i) $(3x^2y^4)^3$

(ii) $\left(\frac{16}{x^{18}y^8}\right)^{-\frac{3}{2}}$

16 Find the n th term of each sequence.

(a) 8, 15, 34, 71, 132,

**ANSWER KEY;**

(b) Expand the brackets and simplify.

$4(3x - 2) - 3(x - 5)$

$$(12x - 8) - (3x - 15)$$

$$12x - 3x - 8 - (-15)$$

$$9x + 7$$

$$\underline{9x + 7} \quad [2]$$

(c) Factorise completely.

$12ab - 20a^2$

$$4a(3b - 5a)$$

$$\underline{4a(3b - 5a)} \quad [2]$$

3 Rearrange the formula to make w the subject.

$5w - 3y + 7 = 0$

$$5w = 3y - 7$$

$$w = \frac{3y - 7}{5}$$

$$w = \underline{\frac{3y - 7}{5}} \quad [2]$$

6 Simplify.

$$\frac{2x^2 + x - 15}{ax + 3a - 2bx - 6b}$$

$$2x^2 + 6x - 5x - 15$$

$$= 2x(x+3) - 5(x+3)$$

$$= (2x - 5)(x + 3)$$

$$ax + 3a - 2bx - 6b$$

$$= a(x+3) - 2b(x+3)$$

$$= (a - 2b)(x + 3)$$

$$\frac{(2x - 5)(x + 3)}{(a - 2b)(x + 3)}$$

$$= \frac{2x - 5}{a - 2b}$$

$$= \frac{2x - 5}{a - 2b}$$

$$\underline{\frac{2x - 5}{a - 2b}} \quad [5]$$



- (e) Solve the simultaneous equations.
You must show all your working.

By Substitution

$$x - 3y = 13$$

$$-3y = 13 - x$$

$$y = \frac{13 - x}{-3}$$

$$2x^2 - 9\left(\frac{13 - x}{-3}\right) = 116$$

$$2x^2 + 39 - 3x = 116$$

$$2x^2 - 3x - 77 = 0$$

$$a = 2 \quad b = -3 \quad c = -77$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x - 3y = 13$$

$$2x^2 - 9y = 116$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-77)}}{2(2)}$$

$$x = 7, -5.5$$

$$y = \frac{13 - 7}{-3}$$

$$y = -2 \quad (7, -2)$$

$$y = \frac{13 - (-5.5)}{-3}$$

$$y = -6.2 \text{ or } -\frac{37}{6} \quad (-5.5, -6.2)$$

$$x = 7 \quad y = -2$$

$$x = -5.5 \quad y = -6.2 \quad [6]$$



- 21 Solve the simultaneous equations.
You must show all your working.

$$4y + 3x = 13$$

$$y = x^2 - 18$$

By elimination

$$y - x^2 = -18$$

$$\boxed{= 4y - 4x^2 = -72}$$

$$4y - 4x^2 = -72$$

$$- 4y + 3x = 13$$

$$-4x^2 - 3x = -72 - 13$$

$$= -4x^2 - 3x = -85$$

$$4x^2 + 3x - 85 = 0$$

$$a = 4 \quad b = 3 \quad c = -85$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(4)(-85)}}{2(4)}$$

$$x = 4.25, -5$$

$$y = 4.25^2 - 18$$

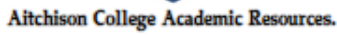
$$y = \frac{1}{16} \text{ or } 0.0625 \quad (4.25, 0.0625)$$

$$y = (-5)^2 - 18$$

$$y = 7 \quad (-5, 7)$$

$$x = 4.25 \quad y = 0.0625$$

$$\text{or } x = -5 \quad y = 7 \quad [5]$$



(a) 8, 15, 34, 71, 132,

7, 19, 37, 61

12, 18, 24

16, 26

	8	15	34	71	132	
.....	n^3	i	8	27	64	125
	7	7	7	7	7	

n^3+7
 n^3+7
[2]

$$Z_{\text{diff}} = n^3$$

(b) $\frac{2}{1}, \xrightarrow{+1} \frac{3}{4}, \xrightarrow{+1} \frac{4}{16}, \xrightarrow{+1} \frac{5}{64}, \xrightarrow{+1} \frac{6}{256}, \dots \rightarrow n+1$

$U_n = ar^{n-1} \rightarrow$ geometric sequence formula

first term $\rightarrow$ common ratio

$$\frac{n+1}{4^{n-1}} \quad [3]$$

$$1 \times 4^{n-1}$$

$$= 4^{n-1}$$

TIP: Solve upper and lower part of fraction separately

(i) $(3x^2y^4)^3$

$$3^3 x^6 y^{12}$$

$$27x^6y^{12} \quad [2]$$

$$\begin{aligned} \text{(ii)} \quad & \left(\frac{16}{x^{16}y^8} \right)^{\frac{3}{2}} \\ & \left(\frac{x^{16}y^8}{16} \right)^{3/2} \\ & = \frac{x^{24}y^{12}}{64} \end{aligned}$$

$$\frac{x^{24} y^{12}}{64}$$



UNIT 3: Coordinate Geometry

3 Coordinate geometry

E3.1 Coordinates

Notes and examples

Use and interpret Cartesian coordinates in two dimensions.

E3.2 Drawing linear graphs

Notes and examples

Draw straight-line graphs for linear equations.

Examples include:

- $y = -2x + 5$
- $y = 7 - 4x$
- $3x + 2y = 5$.

E3.3 Gradient of linear graphs

Notes and examples

- 1 Find the gradient of a straight line.
- 2 Calculate the gradient of a straight line from the coordinates of two points on it.

E3.4 Length and midpoint

Notes and examples

- 1 Calculate the length of a line segment.
- 2 Find the coordinates of the midpoint of a line segment.

E3.5 Equations of linear graphs

Notes and examples

Interpret and obtain the equation of a straight-line graph.

Questions may:

- use and request lines in different forms, e.g.
 $ax + by = c$
 $y = mx + c$
 $x = k$
- involve finding the equation when the graph is given
- ask for the gradient or y-intercept of a graph from an equation, e.g. find the gradient and y-intercept of the graph with equation $5x + 4y = 8$.

Candidates are expected to give equations of a line in a fully simplified form.



3 Coordinate geometry (continued)

E3.6 Parallel lines

Find the gradient and equation of a straight line parallel to a given line.

Notes and examples

e.g. find the equation of the line parallel to $y = 4x - 1$ that passes through $(1, -3)$.

E3.7 Perpendicular lines

Find the gradient and equation of a straight line perpendicular to a given line.

Notes and examples

Examples include:

- find the gradient of a line perpendicular to $2y = 3x + 1$
- find the equation of the perpendicular bisector of the line joining the points $(-3, 8)$ and $(9, -2)$.

Major Terms and associated formulas:

Gradient:

Gradient can be called as **slope** and is represented by a letter **m**.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Formula:

Solved example:

Q: A Line Passes through the points $A(1,2)$ $B(5,6)$.
Find the gradient of this line.

Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6-2}{5-1}$$

$$m = \frac{4}{4}$$

$$\boxed{m = 1}$$

Distance:

It can be known as **length** or **segment** and is represented by **d**.



Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solved Example:

Q: Find the distance / length / segment between point A(1,2) and B(5,6).

Solution

$$d = \sqrt{(6-2)^2 + (5-1)^2}$$

$$d = \sqrt{16 + 16}$$

$$d = \sqrt{32} \quad \text{or } 5.66 \text{ units}$$

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Midpoint:

Formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Solved example:

Case 1:

Q: Find the midpoint of A(1,2) and B(5,6)

Solution

$$\left(\frac{1+5}{2}, \frac{2+6}{2} \right)$$

$$= (3, 4)$$



Case 2:

Q: If midpoint of $P(4,5)$ and $R(x,y)$ is $Q(6,7)$
find coordinates of R

Solution

$$\frac{4+x}{2} = 6$$

$$4+x=12$$

$$x=8$$

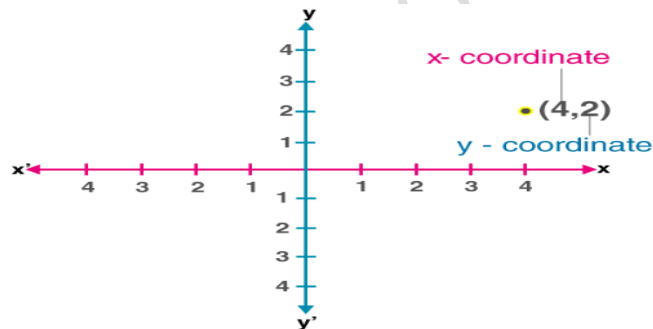
$$\frac{5+y}{2} = 7$$

$$14=5+y$$

$$y=9$$

$$= (8, 9)$$

Cartesian Plane:

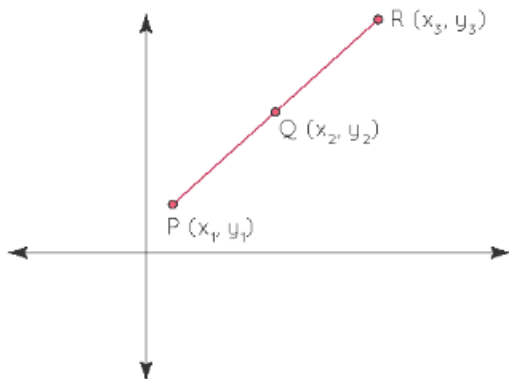


Key points:

“Points are collinear”

Explanation:

All points lie on the same plane/ same line





So, m of $PQ = m$ of $QR = m$ of PR

"Diagonals length are same"

Explanation:

Frequently this sort of concept associated in a question does come where a **quadrilateral shape is made and 3 coordinates of it are given. Then they ask you to find the 4th coordinate.**

First step to solve this question is to figure out whether these shapes(**listed in next bullet point**) are being made by given coordinates or not.

Rectangle, square, parallelogram, kite, trapezium and rhombus are the only quadrilateral whose diagonal lengths are the same.

Hence, if these shapes are identified you can apply this formula:

$$x_1 + x_3 = x_2 + x_4 \quad \text{and} \quad y_1 + y_3 = y_2 + y_4$$

Solved example:

Q1: ABCD is a rectangle $A(-4, -1)$ $B(2, 2)$ $C(6, -6)$
find coordinates of D. (x_4, y_4)

Solution

$$-4 + 6 = 2 + x_4$$

$$x_4 = 0$$

$$-1 + (-6) = 2 + y_4$$

$$y_4 = -9$$

$$D(0, -9)$$

Equation of straight line:

General form : $y = mx + c$

Key:

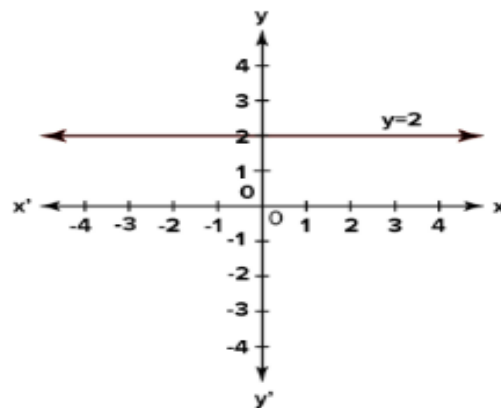
x and y are coordinates of line

m = gradient/slope of line

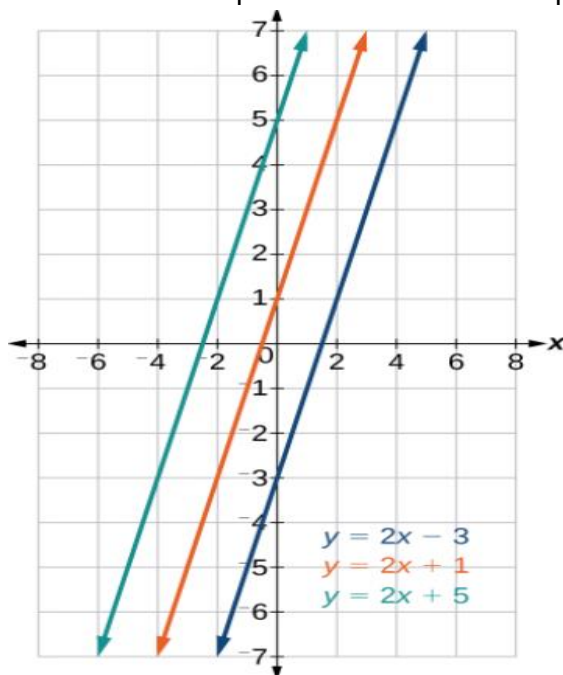
c = y intercept of line

Gradient/slope/m rules in straight line:

When the line is parallel to the x-axis $m = 0$

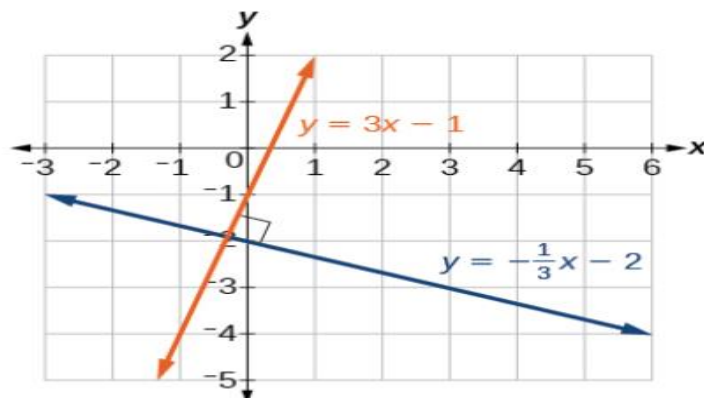


When 2 lines are parallel on the cartesian plane, **both lines gradient/slope/m are equal.**



When 2 lines are perpendicular (intersecting each other at angle of 90), both **line gradients/slope/m whenever multiplied is equal to -1.**

$$(m_1)(m_2) = -1$$





Solved example:

Q1a) A line x passes through $A(-1,1)$ $B(5,5)$. Find equation of line x .

Solution

$$\begin{aligned}\textcircled{1} \text{ m of } x &= \frac{5-1}{5-(-1)} \\ &= \frac{4}{6} \\ &= \frac{2}{3}\end{aligned}$$

$$\boxed{y = \frac{2x+5}{3}}$$

$$\textcircled{2} \quad y = mx + c$$

Insert values of x, y, c
into equation

$$5 = \frac{2}{3}(5) + c$$

$$c = \frac{5}{3}$$

b) A normal line q intersects line x and passes through point $C(4,1)$. Find equation of normal line q .

$$\begin{aligned}\textcircled{1} \text{ m of } q &= \frac{2}{3} m_2 = -1 \\ m_2 &= -\frac{3}{2}\end{aligned}$$

Formula for finding midpoint (m_2)
of the line perpendicular to the
original line is

$$\boxed{m_1 \times m_2 = -1}$$

$\textcircled{2}$ Insert values of x, y and m_2 to find c

$$1 = -\frac{3}{2}(4) + c$$

$$c = 7$$

$$\boxed{y = -\frac{3}{2}x + 7}$$



UNSOLVED PAST PAPER QUESTIONS:

10 (a) A is the point $(6, 2)$ and B is the point $(3, -4)$.

(i) Find the coordinates of the midpoint of AB .

(..... ,) [2]

(ii) Calculate the length AB .

..... [3]

(b) The equation of line l is $-4x + 3y - 12 = 0$.

(i) Find the gradient of l .

..... [2]

(ii) Find the coordinates of the point where l crosses the y -axis.

(..... ,) [2]



ANSWER KEY:

- 10 (a) A is the point $(6, 2)$ and B is the point $(3, -4)$.

(i) Find the coordinates of the midpoint of AB .

$$\begin{aligned} \text{Formula} &= \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) \\ &= \left(\frac{3 + 6}{2}, \frac{-4 + 2}{2} \right) \\ &= \frac{9}{2}, -1 \end{aligned} \quad (4.5, -1) \quad [2]$$

(ii) Calculate the length AB .

$$\begin{aligned} \text{Length} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - 6)^2 + (-4 - 2)^2} \\ &= \sqrt{(-3)^2 + (-6)^2} \\ &= \sqrt{9 + 36} \\ &= \sqrt{45} \end{aligned} \quad 6.71 \quad [3]$$

(b) The equation of line l is $4x + 3y - 12 = 0$.

(i) Find the gradient of l .

x	1	2
y	$8/3$	$4/3$

$$\begin{aligned} 4x + 3y - 12 &= 0 \\ 4(1) + 3y - 12 &= 0 \\ 4 + 3y - 12 &= 0 \\ 3y - 8 &= 0 \\ y &= \frac{8}{3} \end{aligned} \quad \begin{aligned} 4x + 3y - 12 &= 0 \\ 4(2) + 3y - 12 &= 0 \\ 8 - 12 + 3y &= 0 \\ -4 + 3y &= 0 \\ y &= \frac{4}{3} \end{aligned} \quad \begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \left(\frac{4}{3} - \frac{8}{3} \right) \div (2 - 1) \\ m &= \frac{-4}{3} \div 1 \\ m &= -\frac{4}{3} \end{aligned} \quad -4/3 \quad [2]$$



- (ii) Find the coordinates of the point where l crosses the y -axis.

$$50x = 0$$

$$4x + 3y - 12 = 0$$

$$0 + 3y - 12 = 0$$

$$3y = 12$$

$$y = 4$$

$$(0, 4) \quad [2]$$

- (iii) Line p is perpendicular to l and passes through $(6, 5)$.

Find the equation of p in the form $y = mx + c$.

$$m_l = -\frac{4}{3}$$

$$m_p = \frac{-1}{m_l}$$

$$m_p = -1 \div -\frac{4}{3}$$

$$= -1 \times \frac{3}{-4}$$

$$= \frac{3}{4}$$

$$= \frac{3}{4}$$

Formula:

$$y - y_1 = m_p(x - x_1)$$

$$y - 5 = \frac{3}{4}(x - 6)$$

$$4y - 20 = 3x - 18$$

$$4y = 3x - 18 + 20$$

$$4y = 3x + 2$$

$$y = \frac{3}{4}x + \frac{2}{4}$$

$$y = \frac{3}{4}x + \frac{1}{2}$$

$$y = \frac{3}{4}x + \frac{1}{2} \quad [3]$$



UNIT4 : GEOMETRY

4 Geometry

E4.1 Geometrical terms

Notes and examples

1 Use and interpret the following geometrical terms:

- point
- vertex
- line
- plane
- parallel
- perpendicular
- perpendicular bisector
- bearing
- right angle
- acute, obtuse and reflex angles
- interior and exterior angles
- similar
- congruent
- scale factor.

Candidates are **not** expected to show that two shapes are congruent.

2 Use and interpret the vocabulary of:

- triangles
- special quadrilaterals
- polygons
- nets
- solids.

Includes the following terms.

Triangles:

- equilateral
- isosceles
- scalene
- right-angled.

Quadrilaterals:

- square
- rectangle
- kite
- rhombus
- parallelogram
- trapezium.

continued



4 Geometry (continued)

E4.1 Geometrical terms (continued)

Notes and examples

Polygons:

- regular and irregular polygons
- pentagon
- hexagon
- octagon
- decagon.

Solids:

- cube
- cuboid
- prism
- cylinder
- pyramid
- cone
- sphere
- hemisphere
- frustum
- face
- surface
- edge.

3 Use and interpret the vocabulary of a circle.

Includes the following terms:

- centre
- radius (plural radii)
- diameter
- circumference
- semicircle
- chord
- tangent
- major and minor arc
- sector
- segment.



E4.2 Geometrical constructions	Notes and examples
1 Measure and draw lines and angles. 2 Construct a triangle, given the lengths of all sides, using a ruler and pair of compasses only. 3 Draw, use and interpret nets.	A ruler should be used for all straight edges. Constructions of perpendicular bisectors and angle bisectors are not required e.g. construct a rhombus by drawing two triangles. Construction arcs must be shown. Examples include: • draw nets of cubes, cuboids, prisms and pyramids • use measurements from nets to calculate volumes and surface areas.
E4.3 Scale drawings	Notes and examples
1 Draw and interpret scale drawings. 2 Use and interpret three-figure bearings.	A ruler must be used for all straight edges. Bearings are measured clockwise from north (000° to 360°). e.g. find the bearing of <i>A</i> from <i>B</i> if the bearing of <i>B</i> from <i>A</i> is 025°. Includes an understanding of the terms north, east, south and west. e.g. point <i>D</i> is due east of point <i>C</i>.
E4.4 Similarity	Notes and examples
1 Calculate lengths of similar shapes. 2 Use the relationships between lengths and areas of similar shapes and lengths, surface areas and volumes of similar solids. 3 Solve problems and give simple explanations involving similarity.	Includes use of scale factor, e.g. $\frac{\text{Volume of } A}{\text{Volume of } B} = \frac{(\text{Length of } A)^3}{(\text{Length of } B)^3}$ Includes showing that two triangles are similar using geometric reasons.
E4.5 Symmetry	Notes and examples
1 Recognise line symmetry and order of rotational symmetry in two dimensions. 2 Recognise symmetry properties of prisms, cylinders, pyramids and cones.	Includes properties of triangles, quadrilaterals and polygons directly related to their symmetries. e.g. identify planes and axes of symmetry.



4 Geometry (continued)

E4.6 Angles

Notes and examples

- 1 Calculate unknown angles and give simple explanations using the following geometrical properties:
 - sum of angles at a point = 360°
 - sum of angles at a point on a straight line = 180°
 - vertically opposite angles are equal
 - angle sum of a triangle = 180° and angle sum of a quadrilateral = 360° .
- 2 Calculate unknown angles and give geometric explanations for angles formed within parallel lines:
 - corresponding angles are equal
 - alternate angles are equal
 - co-interior angles sum to 180° (supplementary).
- 3 Know and use angle properties of regular and irregular polygons.

Knowledge of 3-letter notation for angles is required, e.g. angle ABC . Candidates are expected to use the correct geometrical terminology when giving reasons for answers.

Includes exterior and interior angles, and angle sum.

E4.7 Circle theorems I

Notes and examples

Calculate unknown angles and give explanations using the following geometrical properties of circles:

- angle in a semicircle = 90°
- angle between tangent and radius = 90°
- angle at the centre is twice the angle at the circumference
- angles in the same segment are equal
- opposite angles of a cyclic quadrilateral sum to 180° (supplementary)
- alternate segment theorem.

Candidates are expected to use the geometrical properties listed in the syllabus when giving reasons for answers.

E4.8 Circle theorems II

Notes and examples

Use the following symmetry properties of circles:

- equal chords are equidistant from the centre
- the perpendicular bisector of a chord passes through the centre
- tangents from an external point are equal in

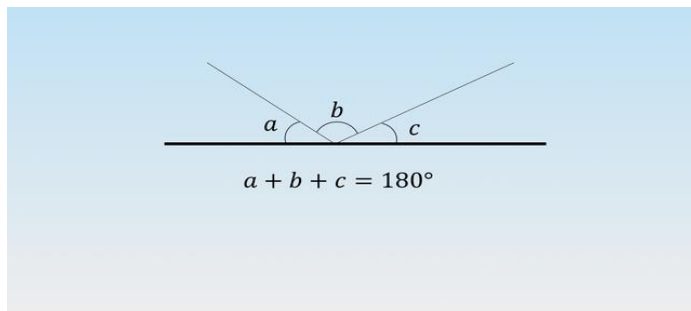
Candidates are expected to use the geometrical properties listed in the syllabus when giving reasons for answers.



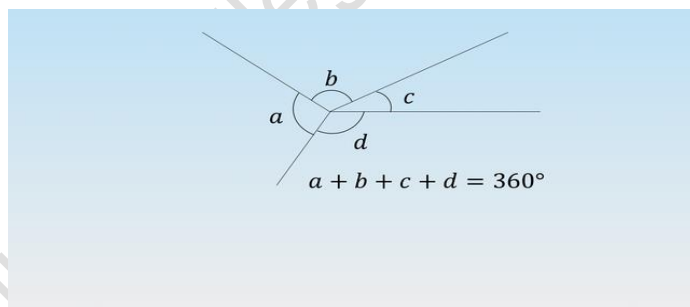
Necessary terms, concepts and formulas

Angles: Appearing in most IGCSE Geometry based questions, angles are formed when two straight lines intersect each other at a point and the corresponding angular distance between those two lines is called an 'angle'. Some of the rules relating to angles are shown below.

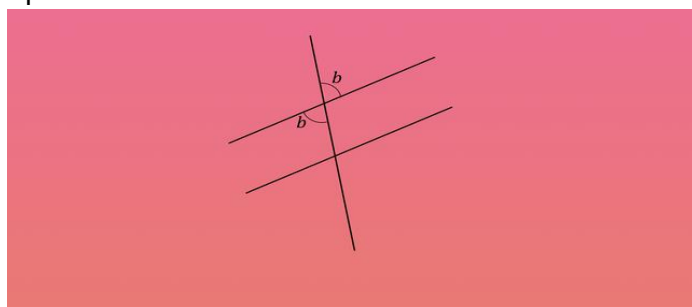
Angles on a straight line always add up to 180°



Angles at point always add up to 360°



Vertically opposite angles are always equal to each other



The sum of all angles of any triangle add up to 180°

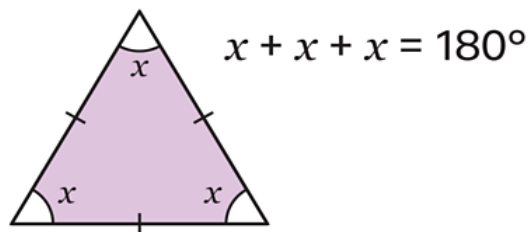
Side Note:

An Isosceles triangle has 2 equal sides and 2 equal angles

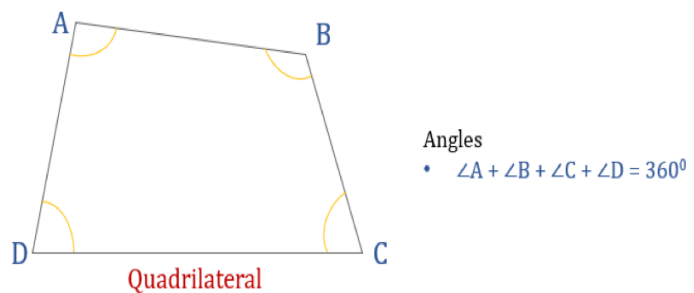
An Equilateral triangle has 3 equal sides and 3 equal angles

A Scalene triangle has 0 equal sides and 0 equal angles

A Right-Angled triangle has one angle which is 90° (it will be talked in more detail in later pages)

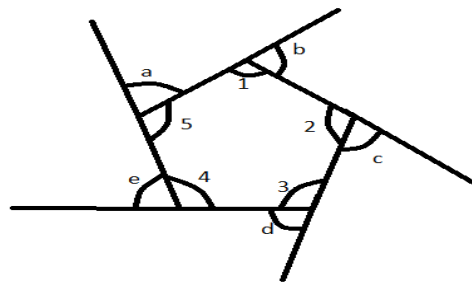


The sum of all angles in a quadrilateral is 360°



Polygons: A closed 2-Dimensional shape formed when straight line segments meet at their endpoints. The line segments are called **sides**, each endpoint is called a **vertex**. The formulas for each type of question appearing in IGCSE examinations relating to Polygons are shown below.

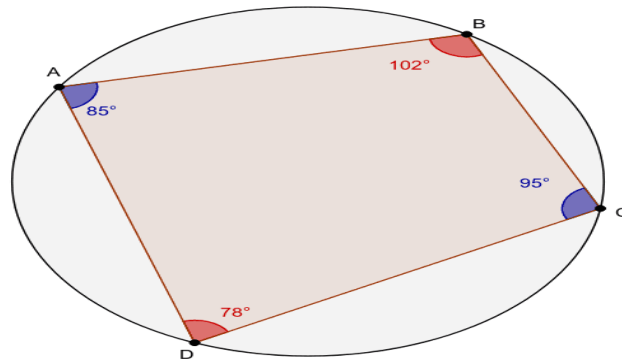
The **exterior** angles of a polygon add up to 360° ($a^\circ + b^\circ + c^\circ + d^\circ + e^\circ = 360^\circ$)



The sum of the **interior** angles of a polygon is denoted by the formula ' $(n-2) \times 180$ ' where n = number of sides of the polygon. In a regular polygon all sides are equal.

Circle Theorems: The properties that illustrate the relationship between angles and the shapes residing inside a circle. Key properties that will be tested on your IGCSE examinations are listed below.

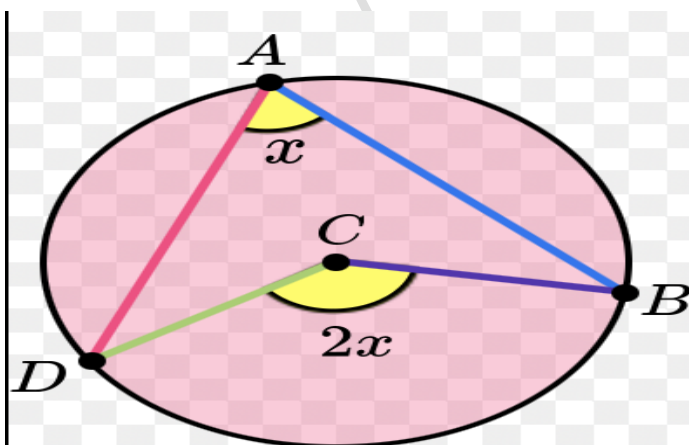
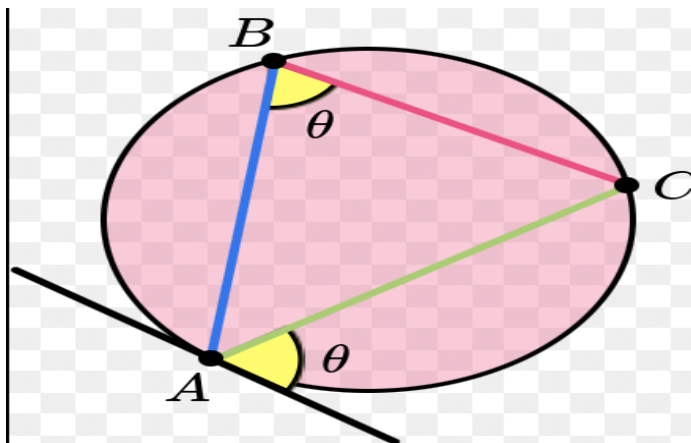
Cyclic quadrilateral: The **opposite** angles within a quadrilateral which lies on the circumference of a circle always add up to 180°



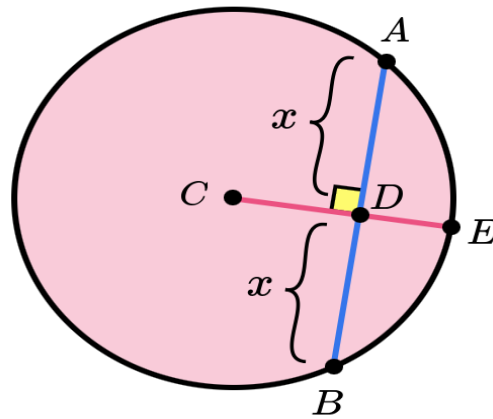
Opposite angles in a cyclic quadrilateral should always total 180°
 Drag the points to alter the angles and illustrate the theory.

Alternate segment theorem: The angle that lies between the tangent and the chord is equal to the angle subtended by the same chord in the alternate segment

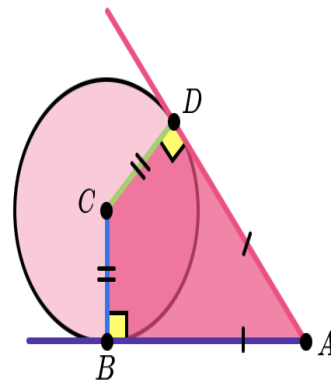
Angle at the center theorem: The angle at the center is twice the angle at the circumference



Chord of a circle: The perpendicular from the center of the circle to a chord bisects the chord making an angle of 90°

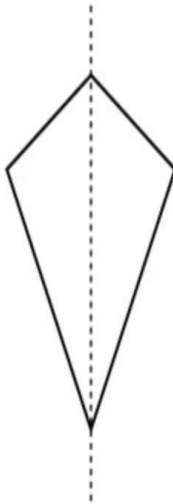
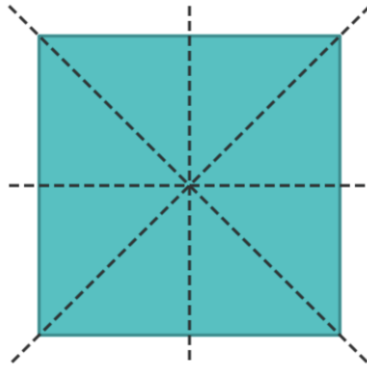


Tangent to a circle: The angle between the tangent to the circle and the radius is always 90° . Both Line AB and AD are tangent to the circle and form 90° angles with the radius



Symmetry: There will be two types of questions that will be tested in your IGCSE examinations: **line of symmetry**, and **order of rotational symmetry**

Line of symmetry: A line of symmetry divides a shape or an object into equal parts. Examples can be seen in multiple geometric shapes such as a square or a kite etc. The kite can only be divided into 2 equal parts. Therefore, It has 1 line of symmetry. On the other hand, a square can be cut into 4 equal parts. Hence, it has 4 lines of symmetry.



Rotational symmetry: The order of rotational symmetry that a shape is the of times it coincides with itself as you rotate it through 360° . Examples include the Rectangle which has a rotational symmetry of 2. The Trapezium with a rotational symmetry of 1etc.

Similarity: Shapes which are similar in the sense that they have the same angles, and therefore, have the same ratio of side lengths as well. This allows us to calculate side lengths of similar triangles as well as the area and volume of said shapes. For example, triangle ABC and DEF are similar. With AB being similar to DE and BC to EF. Then the ratio $AB/DE = BC/EF$ can be used to find 1 of the side lengths if the other 3 are given.

Congruency: Triangles are congruent when they have the same size, and corresponding angle measures and side length are congruent. The 4 types of congruency in triangles is shown below.

SSS(Side-Side-Side): 3 sides are respectively equal.

SAS(Side-Angle-Side): 2 sides are equal and the angle between the two sides should be equal.

ASA(Angle-Side-Angle): 2 angles are equal and the side length between them should be equal.

RHS(Right angle-Hypotenuse-Side): Hypotenuse and one side should be equal.



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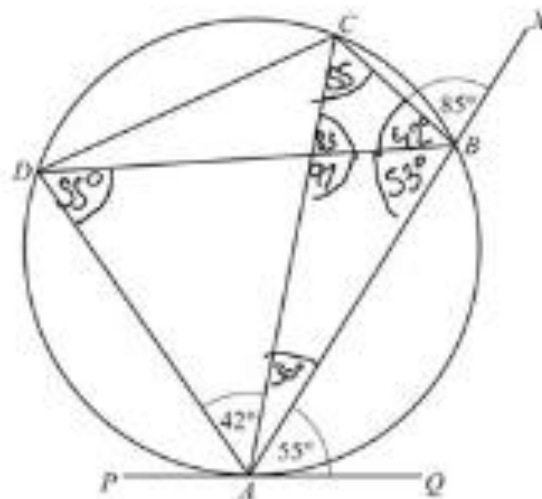
UNSOLVED PAST PAPER QUESTIONS:

- 12 In a regular polygon, the interior angle and the exterior angle are in the ratio interior : exterior = 11 : 1.
Find the number of sides of this regular polygon.
-

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17



NOT TO
SCALE

$ABCD$ is a cyclic quadrilateral, ABX is a straight line and PQ is a tangent to the circle at A .
Angle $CBX = 85^\circ$, angle $BAQ = 55^\circ$ and angle $CAD = 42^\circ$.

Find

(a) angle CBD

55°

(b) angle ACB

42°

(c) angle ADC

55°

(d) angle BCD

(e) angle PAD

1°



re

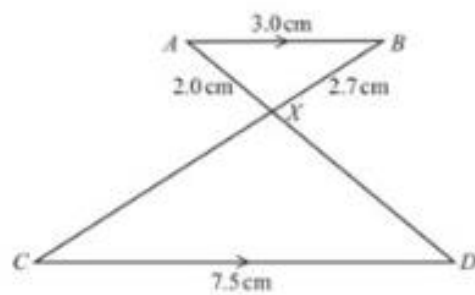
- 18 Two solids are mathematically similar and have volumes 81 cm^3 and 24 cm^3 .
The surface area of the smaller solid is 44 cm^2 .

Calculate the surface area of the larger solid.

..... cm^2 [3]



10



NOT TO
SCALE

In the diagram, AB and CD are parallel.
The lines CB and AD intersect at X .
 $AB = 3.0$ cm, $AX = 2.0$ cm, $BX = 2.7$ cm and $CD = 7.5$ cm.
Find the length of BC .



ANSWER KEY:

- 12 In a regular polygon, the interior angle and the exterior angle are in the ratio interior : exterior = 11 : 1.

Find the number of sides of this regular polygon.

$$\text{interior} + \text{exterior} = 180^\circ$$

$$180 = 11x + 1x$$

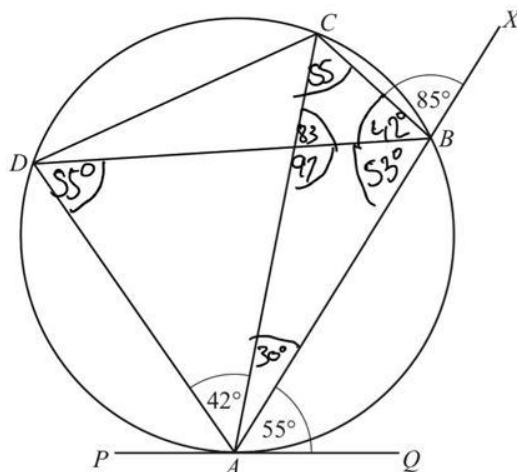
$$\frac{180}{12} = x$$

$$x = 15^\circ$$

$$\text{number of sides} \times 15 = 360$$

$$\frac{360}{15} = 24$$

.....24..... [3]



NOT TO
SCALE

$ABCD$ is a cyclic quadrilateral, ABX is a straight line and PQ is a tangent to the circle at A . Angle $CBX = 85^\circ$, angle $BAQ = 55^\circ$ and angle $CAD = 42^\circ$.

Find

(a) angle CBD

Apply Similar triangles rule

Angle $CBD = 42^\circ$ [1]

(b) angle ACB

Apply Alternate segment theorem

Angle $ACB = 55^\circ$ [1]

(c) angle ADC

$$180 - (42 + 53) = 85$$

Angle $ADC = 85^\circ$ [1]

(d) angle BCD

$$180 - (42 + 30) = 108$$

Angle $BCD = 108^\circ$ [2]

(e) angle PAD .

$$180 - (55 + 30 + 42) = 53$$

Angle $PAD = 53^\circ$ [1]



8

- 18 Two solids are mathematically similar and have volumes 81 cm^3 and 24 cm^3 .
The surface area of the smaller solid is 44 cm^2 .

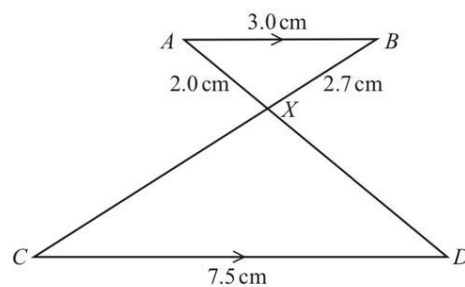
Calculate the surface area of the larger solid.

$$\left(\frac{81}{24}\right) = \left(\frac{x}{y}\right)^3$$
$$\sqrt[3]{\frac{81}{24}} = \frac{3}{2}$$
$$\left(\frac{3}{2}\right)^2 = \left(\frac{x}{44}\right)$$
$$\frac{9}{4} = \frac{x}{44}$$
$$\boxed{x = 99}$$

..... cm^2 [3]



10



NOT TO
SCALE

In the diagram, AB and CD are parallel.
The lines CB and AD intersect at X .
 $AB = 3.0$ cm, $AX = 2.0$ cm, $BX = 2.7$ cm and $CD = 7.5$ cm.

Find the length of BC .

$$\frac{7.5}{3} = \frac{x}{2.7}$$

$$3x = 20.25$$

$$x = 6.75$$

$$x = CX$$

$$CX + BX = BC$$

$$6.75 + 2.7 = 9.45$$

$$BC = 9.45 \text{ cm [3]}$$



UNIT 5: Mensuration

5 Mensuration

E5.1 Units of measure

Use metric units of mass, length, area, volume and capacity in practical situations and convert quantities into larger or smaller units.

Notes and examples

Units include:

- mm, cm, m, km
- mm^2 , cm^2 , m^2 , km^2
- mm^3 , cm^3 , m^3
- mL, l
- g, kg.

Conversion between units includes:

- between different units of area, e.g. $\text{cm}^2 \leftrightarrow \text{m}^2$
- between units of volume and capacity, e.g. $\text{m}^3 \leftrightarrow$ litres.

E5.2 Area and perimeter

Carry out calculations involving the perimeter and area of a rectangle, triangle, parallelogram and trapezium.

Notes and examples

Except for the area of a triangle, formulas are **not** given.

E5.3 Circles, arcs and sectors

- 1 Carry out calculations involving the circumference and area of a circle.
- 2 Carry out calculations involving arc length and sector area as fractions of the circumference and area of a circle.

Notes and examples

Answers may be asked for in terms of π .
Formulas are given in the List of formulas.

Includes minor and major sectors.

E5.4 Surface area and volume

Carry out calculations and solve problems involving the surface area and volume of a:

- cuboid
- prism
- cylinder
- sphere
- pyramid
- cone.

Notes and examples

Answers may be asked for in terms of π .
The following formulas are given in the List of formulas:

- curved surface area of a cylinder
- curved surface area of a cone
- surface area of a sphere
- volume of a prism
- volume of a pyramid
- volume of a cylinder
- volume of a cone
- volume of a sphere.



Notes

Units of Measure

The metric system is the international standard for assigning units to different quantities to measure them. The table below shows these quantities and the different units used to measure them, as well as the prefixes commonly used for each:

Quantity	Units	Common Prefixes
Mass	g (gram)	k (kilo)
Length	m (metre)	m (milli), c (centi), k (kilo)
Area	m ² (square metre)	m (milli), c (centi), k (kilo)
Volume	m ³ (cubic metre)	m (milli), c (centi)
Volume/Capacity	l (litre)	m (milli)

Note: a cubic centimetre is the same value as a millilitre.

The following table shows the indices represented by each prefix:

Prefix	Indice Factor
m (milli)	10^{-3} (1/1000)
c (centi)	10^{-2} (1/100)
k (kilo)	10^3 (1,000)

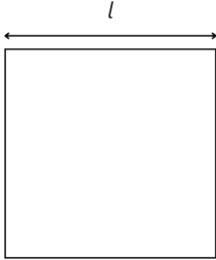
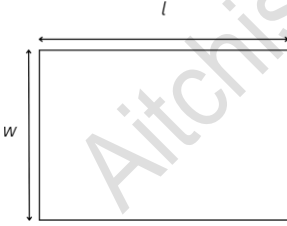
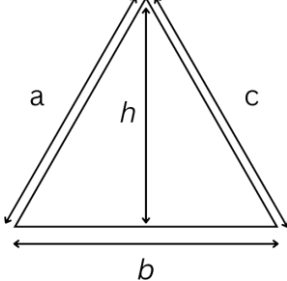
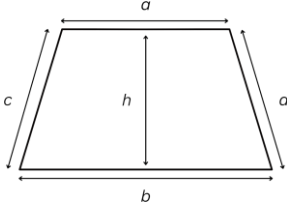
To convert from one unit to another, these factors are taken into account. To convert from a smaller unit to a bigger one, the value is divided, whereas to convert from a bigger unit to a small one, the value is multiplied. For example, to convert from centimetres to kilometres, the value is divided by 100,000. When dealing with square or cubic values, the factor is squared or cubed



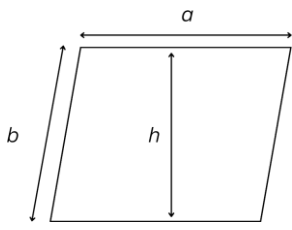
respectively. For example, to convert from cubic metres to cubic millimetres, the value is multiplied by 1000^3 .

Area and Perimeter

The area of a 2-D shape refers to the space that the shape takes up. The perimeter of a 2-D shape refers to the length of the boundary. The following table shows some shapes and the formulas used to find their areas/perimeters.

Shape	Diagram	Formula for Area	Formula for Perimeter
Square		l^2	$4 \times l$
Rectangle		$l \times w$	$2l + 2w$
Triangle		$\frac{1}{2} \times b \times h$	$a + b + c$
Trapezium		$\frac{1}{2} \times h \times (a + b)$	$a + b + c + d$



Parallelogram		$a \times h$	$2 \times a + 2 \times b$
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Circles, Arcs, and Sectors

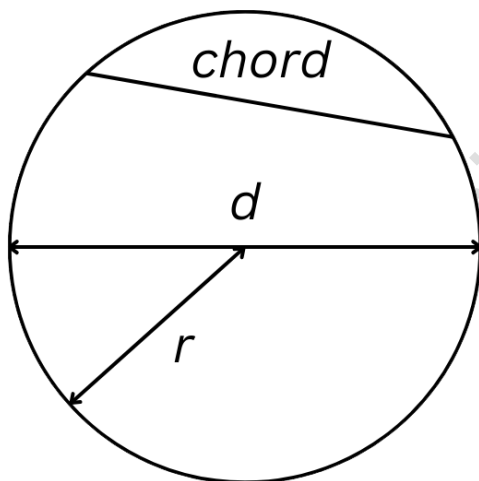
Circumference: The circumference of a circle is its curving perimeter.

Diameter: The distance from one point on a circle's circumference to another, and passing through the centre of the circle. (Denoted by d)

Radius: The distance from any point on the circumference to the centre. (Denoted by r)

Chord: A line connecting any two points of a circumference.

π : A constant that represents the ratio of any circle's circumference to its diameter. $\pi \approx 3.142$.



A circle with key features labelled

Formula for circumference of a circle: $2\pi r$

Formula for area of a circle: πr^2

Semicircle: A half-circle, as illustrated below



A semicircle



Formula for circumference of a semicircle: πr

Formula for perimeter of a semicircle: $\pi r + 2r$

Formula for area of a semicircle: $\frac{1}{2} \times \pi r^2$

Arc: A part of the circumference of a circle.

Two points on a circumference of a circle will create two arcs:

The smaller arc is known as the minor arc

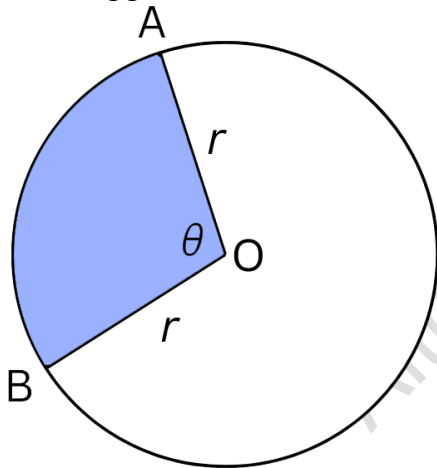
The bigger arc is known as the major arc

Sector: A sector is the part of a circle enclosed by two radii (radiuses) and an arc.

Two radii in a circle will create two sectors:

The smaller sector is known as the minor sector

The bigger sector is known as the major sector



A circle with a marked sector in blue

In the above diagram, the minor sector is the red area and the blue area, whereas the segment is just the blue area. The minor arc is the arc connected to the blue area, whereas the major arc is the rest of the circumference.

The area of any minor sector with angle theta (θ) is $\frac{\theta \pi r^2}{360^\circ}$.

The arc length of any arc with angle theta bound to its corresponding radii is $\frac{\theta \pi r}{180^\circ}$.

The perimeter of any sector with angle theta is $\frac{\theta \pi r}{180^\circ} + 2r$.

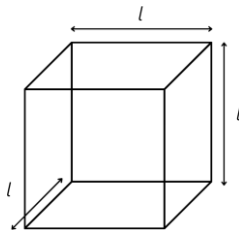
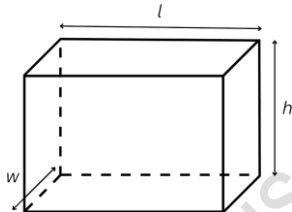
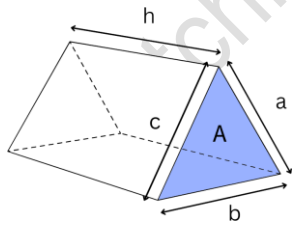
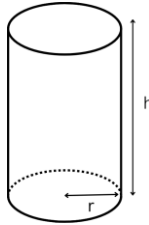
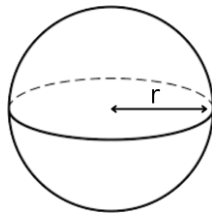
Volume and Surface Area

Volume: The space taken up by a 3-D object. The three dimensions are length, width/breadth, and height/depth.

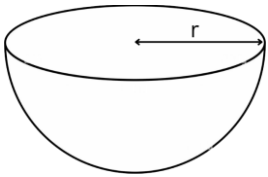
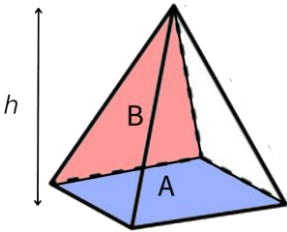
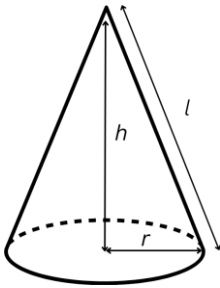


Surface Area: The total area of each face (flat or curved surface) of a 3-D object.

The following table shows some 3-D shapes with the formulas used to calculate their volumes and surface areas.

Shape	Diagram	Formula for Volume	Formula for Surface Area
Cube		l^3	$6l^2$
Cuboid		lwh	$2lw+2lh+2wh$
Prism		Ah	$2A+h(a+b+c+...)$
Cylinder		$\pi r^2 h$	$2\pi r^2 + 2\pi rh$
Sphere		$\frac{4\pi r^3}{3}$	$4\pi r^2$



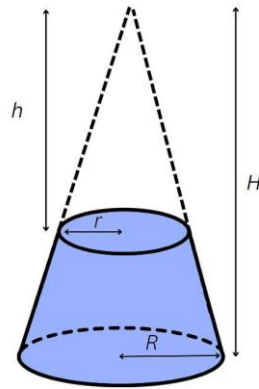
Shape	Diagram	Formula for Volume	Formula for Surface Area
Hemisphere		$\frac{2\pi r^3}{3}$	$3\pi r^2$
Regular Pyramid (With n faces)		$\frac{Ah}{3}$	$A+B(n-1)$
Cone		$\frac{\pi r^2 h}{3}$	$\pi r^2 + \pi r l$

The term prism refers to any solid with a uniform cross-section.

The bases of prisms and pyramids can be any 2-D shape.

The **curved** surface area of a cylinder is $2\pi rh$, and for a cone, it is πrl .

Compound shapes: These are 2-D or 3-D shapes that are formed by joining multiple 2-D or 3-D shapes together. Compound 2-D and 3-D shapes can also be formed by 'cutting' a shape out of another shape. Their respective perimeters and areas (for 2-D shapes), and surface areas and volumes (for 3-D shapes), can be calculated by identifying the original basic shapes used.



A frustum is a compound 3-D shape that is made by cutting a small cone from a larger cone. In this case, a cone with radius r and height h is cut from a larger cone with radius R and height H to form a frustum. For 2-D shapes, there may be a common internal perimeter that should not be considered, and for 3-D shapes, there may be a common internal surface area that should not be considered either. Only external perimeters/surface areas are to be considered.

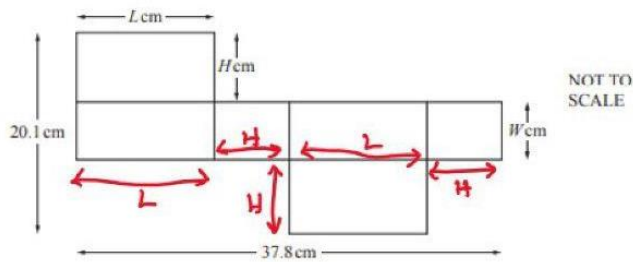
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Questions

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- 8 (a) A cuboid has length L cm, width W cm and height H cm.



The diagram shows the net of this cuboid.

The ratio $W:L = 1:2$.

Find the value of L , the value of W and the value of H .

$$H + W + H = 20.1$$

$$2H + W = 20.1$$

$$2H = 20.1 - W$$

$$W:L$$

$$1:2$$

$$H = \frac{20.1 - 5.9}{2}$$

$$L = 2W$$

$$H = 7.1$$

$$L = 2(5.9)$$

$$L = 11.8$$

$$L + H + L + H = 37.8$$

$$2L + 2H = 37.8$$

$$2(2W) + 2H = 37.8$$

$$4W + 2H = 37.8$$

$$4W + 20.1 - W = 37.8$$

$$3W = 17.7$$

$$W = 5.9$$

$$L = 11.8$$

$$W = 5.9$$

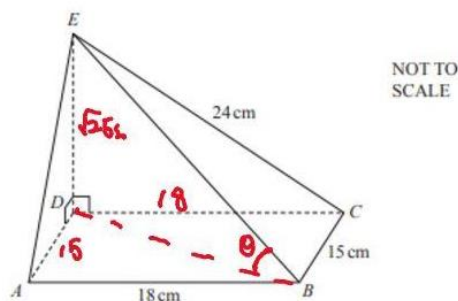
$$H = 7.1$$

[5]



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(b)



The diagram shows a solid pyramid with a rectangular base $ABCD$.

E is vertically above D .

Angle $EDC = \text{angle } EDA = 90^\circ$.

$AB = 18 \text{ cm}$, $BC = 15 \text{ cm}$ and $EC = 24 \text{ cm}$.

(i) The pyramid is made of wood and has a mass of 800 g.

Calculate the density of the wood.

Give the units of your answer.

[The volume, V , of a pyramid is $V = \frac{1}{3} \times \text{area of base} \times \text{height}$.]

[Density = mass $\div$ volume]

$$V = \frac{1}{3} \times (18)(15) \times h$$

$$ED^2 + 18^2 = 24^2$$

$$V = \frac{1}{3} \times 18 \times 15 \times \sqrt{252}$$

$$ED = \sqrt{24^2 - 18^2}$$

$$V = 1482.71$$

$$h = ED = \sqrt{252}$$

$$\text{density} = \frac{800 \text{ g}}{1482.71 \text{ cm}^3} = 0.56 \text{ g/cm}^3$$

(ii) Calculate the angle between BE and the base of the pyramid.

$$DB^2 = 18^2 + 15^2$$

$$\tan \theta = \frac{\sqrt{252}}{\sqrt{549}}$$

$$DB = \sqrt{549}$$

$$\theta = \tan^{-1}(\text{Ans})$$

$$\theta = 34$$

Q8a) - The distance marked as 20.1 cm comprises the width of the cuboid from one of its faces, and two instances of the height of the cuboid from two separate faces. Hence, $2H + W = 20.1 \text{ cm}$. Similarly, the distance marked as 37.8 cm comprises two lengths and two heights. They are heights because once the net is folded, these lengths will be vertically upright. So $2H + 2L = 37.8 \text{ cm}$. Given the ratio $W:L = 1:2$, we know that $L = 2W$. So $2H + 2(2W) = 37.8 \text{ cm}$. $2H + 4W = 37.8 \text{ cm}$. Subtracting the other equation from this, we get



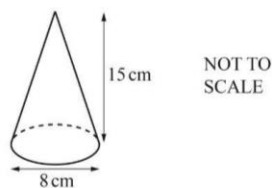
$3W = 37.8 - 20.1$ cm, so $3W = 17.7$. This gives W as 5.9 cm. Substituting this value back into the first equation, we can find $2H$, and then H as 7.1 cm. Now using this value of H , we can find $2L$ and thus L , and find it to be 11.8 cm.

b) i) - We can calculate the base area as the area of an 18×15 cm rectangle. This yields 270 cm^2 . Since EDC is a right triangle, the height ED can be calculated as the square root of the difference of the squares of 24 and 18. This gives the height as $\sqrt{252}$ cm. We can now use this into the given formula to find the volume as approximately 1480 cubic centimetres. Density is mass divided by volume, so in this case, it would be $800\text{g}/1480\text{cm}^3$, giving 0.541g/cm^3 .

b) ii) - We want to find angle EBD . We know that BD is the diagonal of the base rectangle, so it would be the hypotenuse of a right triangle with sides 15 and 18. Thus, BD would be the root of the sum of the squares of 15 and 18, yielding $\sqrt{549}$ cm. We know the height of the right triangle, EBD , which is the root of 26. Knowing the base and the height, we can use the trigonometric ratio of $\tan$, which is the opposite over adjacent, to calculate the value of the tangent of the angle we want to find. This gives $\tan \theta$ as $\sqrt{252/549}$. Using the $\tan$ inverse function, also known as the $\arctan$, we can calculate θ as 34.1 degrees.



5 (a)



A cone has base diameter 8 cm and perpendicular height 15 cm.

- (i) Calculate the volume of the cone.

[The volume, V , of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

$$V = \frac{1}{3}\pi(4)^2 \times 15$$

$$V = 251.33$$

$$\dots\dots\dots 251.33 \text{ cm}^3 \text{ [2]}$$

- (ii) A label completely covers the curved surface area of the cone.

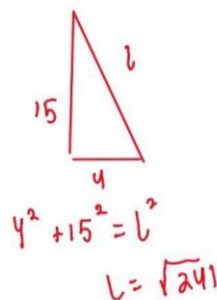
Calculate the area of the label as a percentage of the **total** surface area of the cone.

[The curved surface area, A , of a cone with radius r and slant height l is $A = \pi rl$.]

$$\begin{aligned} \text{C.S.A} &= \pi(4)(\sqrt{241}) \\ &= 195.08 \end{aligned}$$

$$\begin{aligned} \text{Total Area} &= \text{C.S.A} + \text{base Area} \\ &= 195.08 + \pi(4)^2 \\ &= 245.35 \end{aligned}$$

$$\frac{195.08}{245.35} \times 100\% = 79.5$$

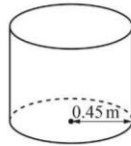


$$\dots\dots\dots 79.5 \text{ \% [5]}$$



11

(b)



NOT TO
SCALE

An empty cylindrical container has radius 0.45 m.
300 litres of water is poured into the container at a rate of 375 ml per second.

(i) Find the time taken, in minutes and seconds, for all the water to be poured into the container.

$$\begin{aligned}
 &375 \text{ ml} : 1 \text{ s} \\
 &300000 \text{ ml} : x \text{ s} \quad x = 800 \text{ s} \rightarrow 13 \text{ min } 20 \text{ s} \\
 &x = \frac{300000}{375} \\
 &\quad \quad \quad 13 \text{ min } 20 \text{ s [3]}
 \end{aligned}$$

(ii) Calculate the height of the water in the container.

$$\begin{aligned}
 &300 \text{ l} = 0.3 \text{ m}^3 \\
 &V = \pi r^2 \times h \\
 &0.3 = \pi (0.45)^2 \times h \\
 &h = \frac{0.3}{\pi \times 0.45^2} \\
 &\quad \quad \quad 0.472 \text{ m [3]}
 \end{aligned}$$

Q5a)i) - The radius is half of the diameter, so if $d = 8 \text{ cm}$, $r = 4 \text{ cm}$. This can be directly plugged into the given formula with the height to give a volume of 251 cm. (Final answer should be given in 3 s.f.)

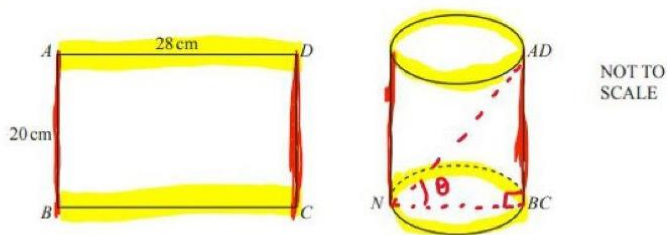
a)ii) - The slant height, also known as l , would be the hypotenuse of the triangle with base r , and height h . So in this case, with $r = 4$ and $h = 15$, l would be the root of the sum of the squares of 4 and 15, giving $l = \text{root of } 241$. This can be plugged into the formula for curved surface area to give it as 195.08. The total area is this curved area summed with the base area which is simply the circle base's area, which is π into the radius squared. This gives a final area of 79.5 cm².

b)i) - 300 litres can be multiplied by 1000 millilitres per litre to give 300,000 millilitres. Dividing this by 375 millilitres per second, we get 800 seconds, which when divided by 60, gives 13.33 minutes. 13 minutes is 780 seconds, so the remainder is 20 seconds. Thus, the time taken to fill the container is 13 mins and 20 secs.

b)ii) - The volume V is 300 litres, which is 300,000 millilitres, which is 300,000 cubic centimetres. This can be divided by 100³ cm³ per m³ to give 0.3 cubic metres as the final volume. Using the formula for volume of a cylinder, and plugging in r , we can find h to be approximately 0.472 metres.



11 (a)



A rectangular sheet of paper $ABCD$ is made into an open cylinder with the edge AB meeting the edge DC .

$AD = 28$ cm and $AB = 20$ cm.

(i) Show that the radius of the cylinder is 4.46 cm, correct to 3 significant figures.

$$\text{Circumference} = 2\pi r$$

$$28 = 2\pi r$$

$$r = \frac{28}{2\pi} \quad r = 4.46$$

[2]

(ii) Calculate the volume of the cylinder.

$$V = \pi r^2 \times h$$

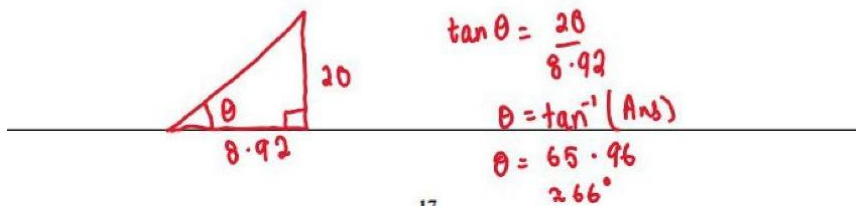
$$V = \pi (4.46)^2 \times 20$$

$$= 1249.83 \approx 1250$$

1250 cm^3 [2]

(iii) N is a point on the base of the cylinder, such that BN is a diameter.

Calculate the angle between AN and the base of the cylinder.



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(b) The volume of a solid cone is 310 cm^3 .
The height of the cone is twice the radius of its base.

Calculate the slant height of the cone.

[The volume, V , of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]



$$V = 310$$

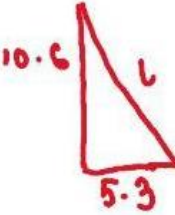
$$\frac{1}{3}\pi r^2 \times 2r = 310$$

$$r^3 = \frac{310 \times 3}{2 \times \pi}$$

$$r^3 = 148.014$$

$$r = 5.3$$



$$\begin{aligned} h &= 2r \\ h &= 2(5.3) \\ h &= 10.6 \end{aligned}$$

$$\begin{aligned} l^2 &= 10.6^2 + 5.3^2 \\ l &= \sqrt{140.45} \\ &= 11.85 \end{aligned}$$

..... 11.85 cm [5]

Q11a)i) - Since AB is meeting DC, we can find that the 28 cm side is the circumference of the cylinder. Using this into $C = 2\pi r$, we can find the radius, r as approximately 4.46 cm.

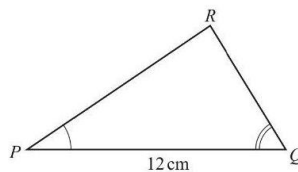
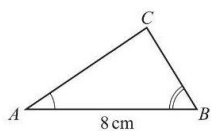
a)ii) - Volume is the product of π , the radius squared, and the height; since we know the height would be AB or DC = 20 cm, we can simply plug in our values to get $V = 1250 \text{ cm}^3$.

iii) - If BN is the diameter, we can then calculate that $BN = 2 \times 4.46 = 8.92 \text{ cm}$. AN is the hypotenuse of a triangle with the height AB and the base BN. So the $\tan$ of the angle we want to find would be AB/BN , and then using $\tan$ inverse, we can find the angle as 66.0 degrees (up to 1 decimal).

b) - Using the information given, we know that $h = 2r$. So in the formula $V = \pi r^2 h$, we can substitute h as $2r$, to get $V = \pi \times 2r^3$. Since $V = 310 \text{ cm}^3$, we can find r to be 5.29 using this formula (up to 3 significant figures). We know that l , the slant height, is the hypotenuse of a triangle with the radius as the base and the height as $2r$. So l is the root of the sum of the squares of 5.3 and 10.6, giving the slant height as approximately 11.9 cm.



8 (a)



NOT TO
SCALE

Triangle ABC is mathematically similar to triangle PQR .
The area of triangle ABC is 16 cm^2 .

- (i) Calculate the area of triangle PQR .

$$K = \frac{12}{8} = 1.5$$

$$K^2 = 2.25$$

$$16 \times 2.25 = 36$$

..... 36 cm^2 [2]

- (ii) The triangles are the cross-sections of prisms which are also mathematically similar.
The volume of the smaller prism is 320 cm^3 .

Calculate the length of the larger prism.

$$K^3 = 3.375$$

$$320 \times 3.375 = 1080$$

$$\frac{1080}{36} = 30$$

..... 30 cm [3]



- (b) A cylinder with radius 6 cm and height h cm has the same volume as a sphere with radius 4.5 cm.

Find the value of h .

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$$V = \frac{4}{3}\pi (4.5)^3$$

$$= 381.7$$

$$\pi r^2 h = 381.7$$

$$h = \frac{381.7}{\pi (6)^2} = 3.375$$

$$h = \dots\dots\dots 3.375 \quad [3]$$

- (c) A solid metal cube of side 20 cm is melted down and made into 40 solid spheres, each of radius r cm.

Find the value of r .

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$$40 \times \frac{4}{3}\pi r^3 = 20^3$$

$$r^3 = \frac{20^3}{40 \times \frac{4}{3}\pi}$$

$$r^3 = 47.7$$

$$r = \sqrt[3]{47.7}$$

$$r = 3.627$$

$$\approx 3.63$$

$$r = \dots\dots\dots 3.63 \quad [3]$$

- (d) A solid cylinder has radius x cm and height $\frac{7x}{2}$ cm.

The surface area of a sphere with radius R cm is equal to the total surface area of the cylinder.

Find an expression for R in terms of x .

[The surface area, A , of a sphere with radius r is $A = 4\pi r^2$.]

$$2\pi x^2 + 2\pi x \left(\frac{7x}{2}\right) = 4\pi R^2$$

$$2x^2 + 7x^2 = 4R^2$$

$$9x^2 = 4R^2$$

$$R^2 = \frac{9x^2}{4}$$

$$R = \sqrt{\frac{9x^2}{4}} = \frac{3}{2}x$$

$$R = \dots\dots\dots \frac{3x}{2} \quad [3]$$

Q8a)i) - We can find the ratio of the sides of the larger triangle to the smaller one as $12/8 = 1.5$. The ratio of the areas would be 1.5^2 , giving 2.25. If the smaller triangle has area 16, the larger one would have area $16 \times 2.25 = 36 \text{ cm}^2$.

a)ii) - The ratio of the volumes would be $1.5^3 = 3.375$. If the smaller prism has volume 320 cm^3 , the larger one would have volume $320 \times 3.375 = 1080 \text{ cm}^3$. Dividing this by the cross sectional area of the prism, which we already found to be 36, we would get the height/length of the prism, depending on its orientation. This is 30 cm.

b) - Since the volumes are equal, we would set the volume of the cylinder as equal to the volume of the sphere. So $\pi \times 6^2 \times h = 4(\pi \times 4.5^3)/3$. Using this, we could then move all the terms except for h to the right hand side, to find h as 3.375 cm.



c) - The cube's volume would be 20^3 , which would be 8000 cm^3 . Dividing this by 40 spheres which would use this total volume, we get the volume of one sphere as 200 cm^3 . Substituting this value of V into the volume formula, we can find r to be 3.63 cm approximately.

d) - Since the surface areas are equal, we would set the surface area of the cylinder as equal to that of the sphere. So $2\pi x^2 + 2\pi x^2 \cdot \frac{7}{2} = 4\pi R^2$. Simplifying the left hand side, we get $9\pi x^2 = 4\pi R^2$. Dividing both sides by 4π , we get $R^2 = \frac{9x^2}{4}$. Taking the square root of both sides, we can get R in terms of x . This gives $R = \frac{3x}{2}$.

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Unit 6: Trigonometry

6 Trigonometry

E6.1 Pythagoras' theorem

Notes and examples

Know and use Pythagoras' theorem.

E6.2 Right-angled triangles

Notes and examples

- 1 Know and use the sine, cosine and tangent ratios for acute angles in calculations involving sides and angles of a right-angled triangle.
- 2 Solve problems in two dimensions using Pythagoras' theorem and trigonometry.
- 3 Know that the perpendicular distance from a point to a line is the shortest distance to the line.
- 4 Carry out calculations involving angles of elevation and depression.

Angles will be given in degrees and answers should be written in degrees, with decimals correct to one decimal place.

Knowledge of bearings may be required.

E6.3 Exact trigonometric values

Notes and examples

Know the exact values of:

- 1 $\sin x$ and $\cos x$ for $x = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° .
- 2 $\tan x$ for $x = 0^\circ, 30^\circ, 45^\circ$ and 60° .

E6.4 Trigonometric functions

Notes and examples

- 1 Recognise, sketch and interpret the following graphs for $0^\circ \leq x \leq 360^\circ$:

- $y = \sin x$
- $y = \cos x$
- $y = \tan x$.

- 2 Solve trigonometric equations involving $\sin x$, $\cos x$ or $\tan x$, for $0^\circ \leq x \leq 360^\circ$.

e.g. solve:

- $\sin x = \frac{\sqrt{3}}{2}$ for $0^\circ \leq x \leq 360^\circ$
- $2 \cos x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.

Notes:

Types of Triangles

Equilateral Triangles

All sides are of equal length

All angles are equal to 60°



Isosceles Triangles

2 sides are of equal length

2 angles are equal

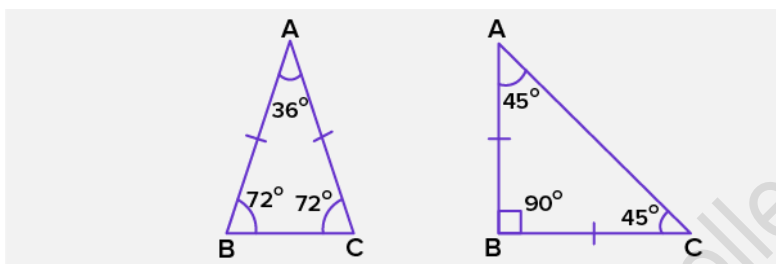
Scalene Triangle

None of the sides are equal

None of the angles are equal

Right-Angled Triangles

One of the angles is equal to 90°



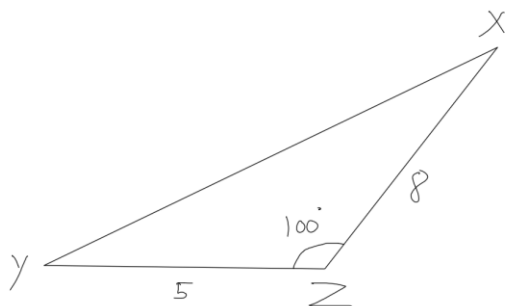
Isosceles Triangles

Area of Triangles

There is one formula which can be used to calculate the area of any triangle.

$$\text{Area} = \frac{1}{2}ab \cdot \sin(\theta)$$

a and b are the 2 sides of a triangle and θ is the angle between those sides.



Area : ?

$$\text{Area} = \frac{1}{2}(5)(8) \sin(100)$$

$$= 20 \sin(100)$$

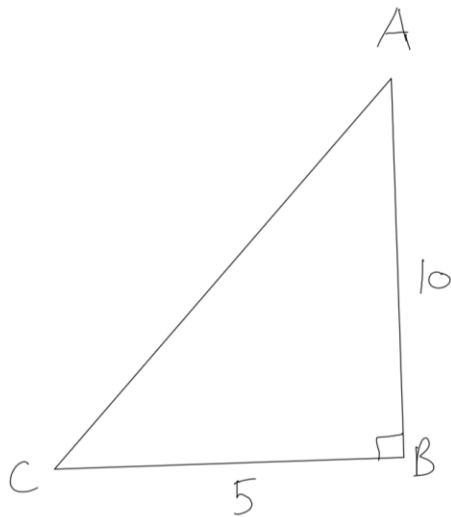
$$= 19.7$$

The formula can be shortened for right angled triangles. If the lengths of the sides of the triangle which make the 90 degree angle are known, the formula can be used as follows:



$$\text{Area} = \frac{1}{2}ab$$

a and b are the 2 sides which make the right angle. $\sin(90)$ is 1 which is not written.



$$\text{Area} = ?$$

$$\frac{1}{2} (5)(10) \sin(90)$$

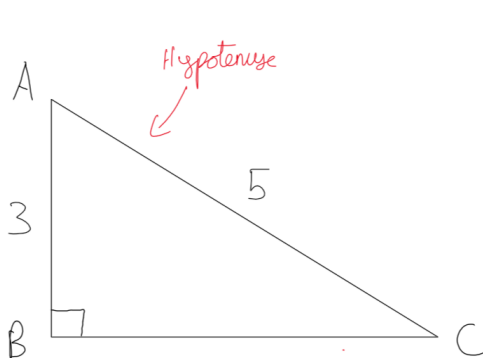
$$\frac{1}{2} (5)(10)$$

$$\text{Area} = 25$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

This theorem relates the lengths of the sides of a right angled triangle. c is the longest side of the triangle (hypotenuse) which is opposite of the right angle. a and b are the other 2 sides.



$$BC = ?$$

$$a^2 + b^2 = c^2$$

$$AB^2 + BC^2 = AC^2$$

$$3^2 + BC^2 = 5^2$$

$$9 + BC^2 = 25$$

$$BC^2 = 16$$

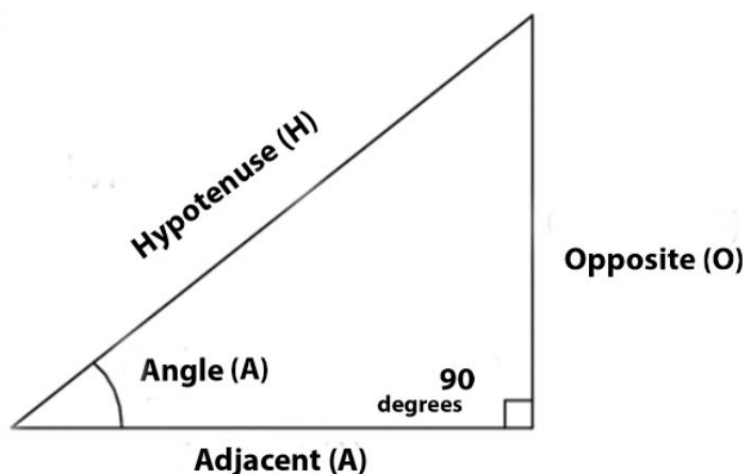
$$BC = \sqrt{16}$$

$$BC = 4$$

Trigonometric Functions



The 3 trigonometric functions which you will study at this level are $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$ where θ is an angle. These functions can only be applied to right angled triangles to find the values of missing sides and angles. Before understanding how to apply them, you need to be able to recognise the different sides of a triangle.



The side opposite to the right angle is always called the hypotenuse. This is the longest side in a right angled triangle. The names of the other 2 sides change depending on which angle is being used in the function. For example, if angle A is being used, the side opposite to that angle will be called opposite. Once you have labelled the hypotenuse and opposite, the remaining side will be the adjacent side. If I take into account the angle at the top of the triangle, the opposite and adjacent sides will be switched.

Sin Opposite Hypotenuse - SOH

$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

Cos Adjacent Hypotenuse - CAH

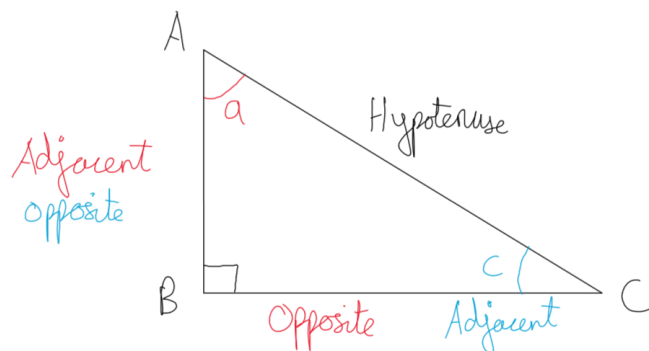
$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

Tan Opposite Adjacent - TOA

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$$



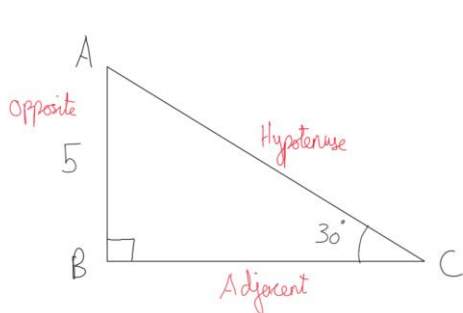
You need to remember this mnemonic to solve questions related to trigonometry. A different function is to be used depending on what side lengths/angles are given and each one is used for a specific scenario. Let's look at an example.



$$\sin(a) = \frac{BC}{AC}, \cos(a) = \frac{AB}{AC}, \tan(a) = \frac{BC}{AB}$$

$$\sin(c) = \frac{AB}{AC}, \cos(c) = \frac{BC}{AC}, \tan(c) = \frac{AB}{BC}$$

The functions in red correspond to the angle a and the functions in blue correspond to the angle c . Here you can also see that the names of sides change depending on which angle is being looked at. Let's look at an example.



$$AC = ?$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\sin(30) = \frac{5}{AC}$$

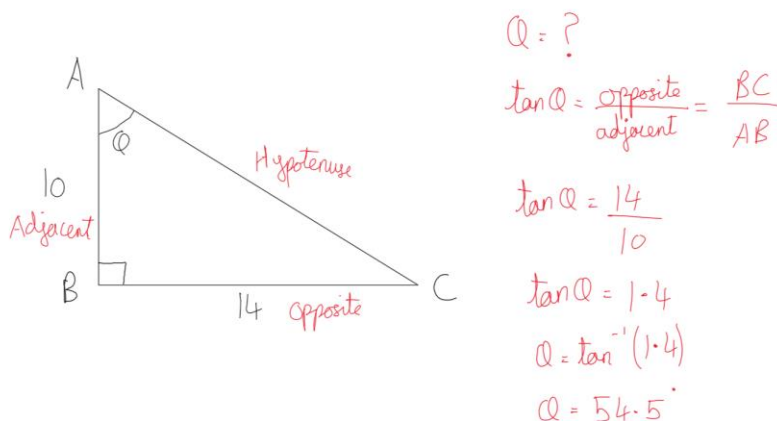
$$\frac{1}{2} = \frac{5}{AC}$$

$$AC = 10$$

Look at the available data and what the question is asking for. Side length AB and angle ACB is given and we need to find side AC. First, label all the sides with respect to the angle ACB (done in red). Now identify which function will be applied. The opposite is known and the hypotenuse needs to be



calculated. The only function relating the opposite and the hypotenuse is sin. Plug in the values in the formula. Use your calculator to find the value of $\sin(30)$ and then find AC.



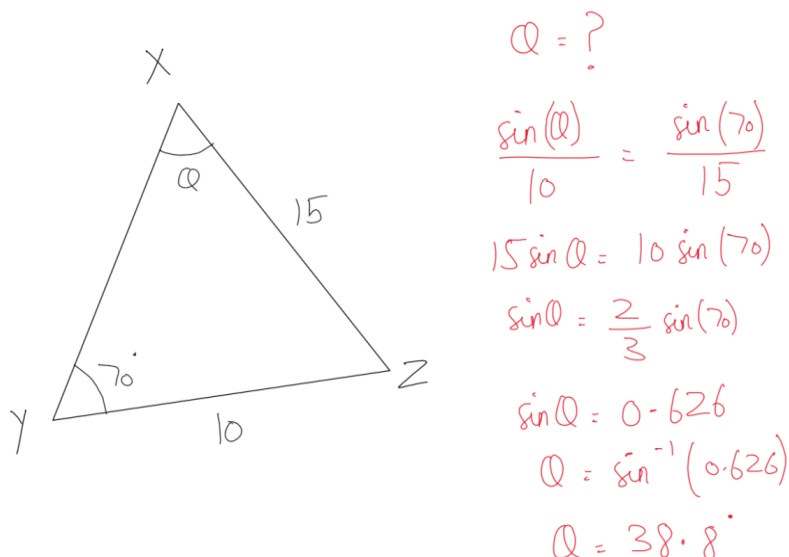
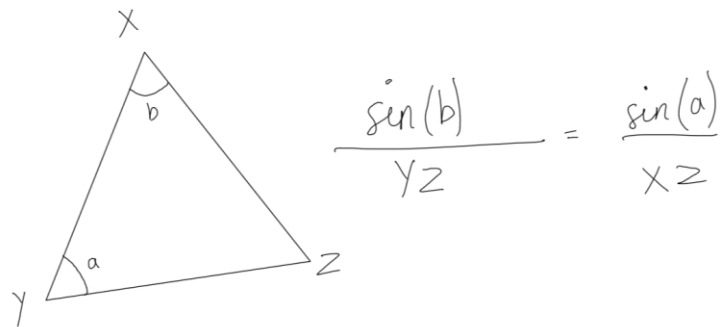
In this example, 2 sides are given and angle needs to be found. Start off by labeling the sides with respect to θ . The 2 known sides are opposite and adjacent. The only function relating opposite and adjacent is tan. To find the value of θ from $\tan(\theta) = 1.4$ you need to make use of inverse functions. Think of it as 'moving' the tan from one side of the equation to the other and in the process, the tan becomes $\tan^{-1}$ (tan inverse). The equation will now be $\theta = \tan^{-1}(1.4)$. Press shift and the tan button on your calculator to get to the tan inverse from where the value of θ can be found. The inverse method works with all 3 functions.

Sine Rule

The sine rule is different from the trigonometric function sin but it incorporates the sin function. It used to find missing sides and angles in non-right angled triangles.

$$\frac{\sin(a)}{A} = \frac{\sin(b)}{B}$$

A and B are sides and a and b are angles. It is important to note that the angle a must be opposite of the side A and the same case with side B and angle b .



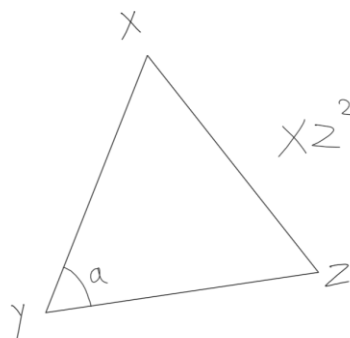
In the above example, 2 sides and an angle are given and angle θ needs to be found. It is apparent that the angles and sides are opposite each other so the sine rule can be applied. θ can now be found.

Cosine Rule

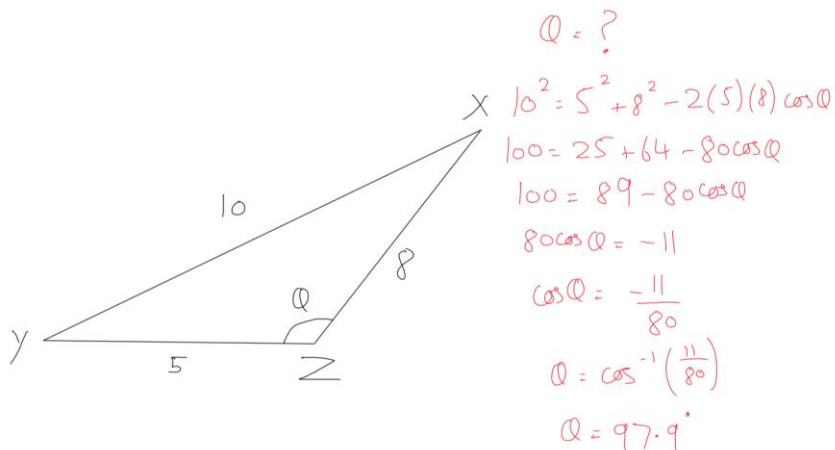
The cosine rule, again, is different from the cos trigonometric identity but it incorporates it into its formula. This is also for non-right angled triangles. You will need to identify whether the sine or cosine rule needs to be applied depending on the known values in the question and what the question is asking for.

$$A^2 = B^2 + C^2 - 2BC \cdot \cos(a)$$

A , B and C are the sides of the triangle and angle a is the angle opposite to side A . This is similar to the sine rule where the angle a and side A need to be opposite each other. Sides B and C are the remaining 2 sides.



$$XZ^2 = XY^2 + YZ^2 - 2(XY)(YZ)\cos(a)$$



$$Q = ?$$

$$10^2 = 5^2 + 8^2 - 2(5)(8)\cos Q$$

$$100 = 25 + 64 - 80\cos Q$$

$$100 = 89 - 80\cos Q$$

$$80\cos Q = -11$$

$$\cos Q = \frac{-11}{80}$$

$$Q = \cos^{-1}\left(\frac{-11}{80}\right)$$

$$Q = 97.9^\circ$$

In this example, it is apparent that the sine rule cannot be applied because no angle is known. The cosine rule must now be applied. Angle θ needs to be found and the side opposite θ is XY. Substitute values in the formula and find θ .

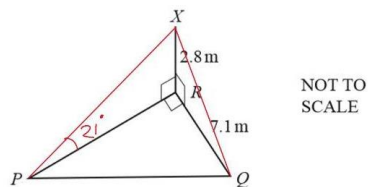


Questions

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7 (a)



The diagram shows a right-angled triangle PQR on horizontal ground. X is vertically above R and the angle of elevation of X from P is 21° . $XR = 2.8$ m and $RQ = 7.1$ m.

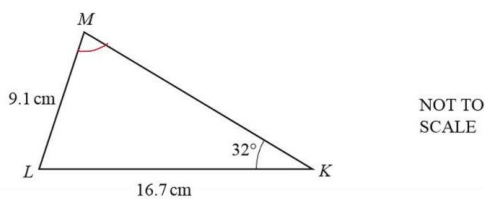
(i) Calculate the angle of elevation of X from Q .

$$\begin{array}{l} \text{Handwritten solution for (i):} \\ \text{Triangle } XQR \text{ is a right-angled triangle with right angle at } R. \\ \text{Side } XR = 2.8, \text{ Side } RQ = 7.1. \\ \tan XQR = \frac{2.8}{7.1} \\ XQR = 21.5^\circ \end{array}$$

(ii) Calculate PQ .

$$\begin{array}{l} \text{Handwritten solution for (ii):} \\ \text{First, find } PR \text{ using triangle } PXR. \\ \tan 21^\circ = \frac{2.8}{PR} \Rightarrow PR = 7.29 \\ \text{Then, use Pythagoras' theorem in triangle } PQR: \\ PQ^2 = PR^2 + RQ^2 \\ PQ^2 = 7.29^2 + 7.1^2 \\ PQ = 10.17 \approx 10.2 \text{ m} \end{array}$$

(b)

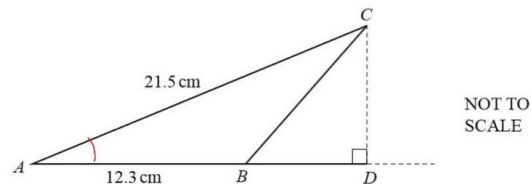


Calculate the acute angle KML .

$$\begin{array}{l} \text{Handwritten solution for (b):} \\ \text{Using Sine Rule:} \\ \frac{\sin 32^\circ}{9.1} = \frac{\sin KML}{16.7} \\ \sin KML = 0.972 \\ KML = 76.529^\circ \approx 76.5^\circ \end{array}$$



(c)



The area of triangle ABC is 62.89 cm^2 .

(i) Show that angle $BAC = 28.4^\circ$, correct to 1 decimal place.

$$\frac{1}{2}(12.3)(21.5)\sin BAC = 62.89$$

$$\sin BAC = \frac{62.89}{132.225}$$

$$BAC = 28.400 \approx 28.4$$

[2]

(ii) Calculate BC .

Using Cosine Rule

$$BC^2 = 12.3^2 + 21.5^2 - 2(12.3)(21.5)\cos(28.4)$$

$$BC^2 = 613.54 - 528.9(0.879)$$

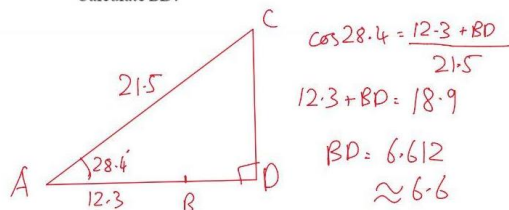
$$BC^2 = 148.29$$

$$BC = 12.17 \approx 12.2$$

$$\dots\dots\dots 12.2 \dots\dots\dots \text{cm} [3]$$

(iii) AB is extended to a point D such that angle $BDC = 90^\circ$.

Calculate BD .



$$\cos 28.4 = \frac{12.3 + BD}{21.5}$$

$$12.3 + BD = 18.9$$

$$BD = 6.612$$

$$\approx 6.6$$

$$\dots\dots\dots 6.6 \dots\dots\dots \text{cm} [3]$$

Q7a(i) - Angle of elevation from X to Q refers to the angle that the line XQ makes with the horizontal base or how 'elevated' the line is, in degrees, from the base. 2 of the lengths are known so tan can be used to find the angle.

Q7a(ii) - Using the vertical triangle XPR, PR can be found. Pythagorean theorem can be used to find the hypotenuse of triangle PQR, PQ.

Q7b - Sine rule can be used to equate the ratios of the sin of the angle to the length opposite to that angle.



Q7c(i) - The formula for the area of a triangle is $\frac{1}{2} \times a \times b \times \sin\theta$ where θ is the angle between the sides a and b . Side lengths and area is known so angle can be found.

Q7c(ii) - Cosine rule can be used here. $a^2 = b^2 + c^2 - 2bc \cos\theta$ where a, b and c are side lengths and θ is the angle opposite to side a .

Q7c(iii) - $AD = 12.3 + BD$. This gives AD in terms of BD . Cos can now be applied to get BD .

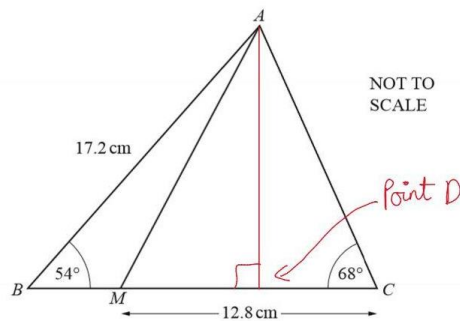
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The diagram shows triangle ABC with $AB = 17.2$ cm.
Angle $ABC = 54^\circ$ and angle $ACB = 68^\circ$.

(a) Calculate AC .

$$\frac{\sin 68}{17.2} = \frac{\sin 54}{AC}$$

$$AC \sin 68 = 13.9$$

$$AC = 15$$

$$AC = 15 \text{ cm [3]}$$

(b) M lies on BC and $MC = 12.8$ cm.

Calculate AM .

$$AM^2 = 15^2 + 12.8^2 - 2(15)(12.8)\cos 68$$

$$AM^2 = 388.84 - 143.8$$

$$AM^2 = 245.04$$

$$AM = 15.65$$

$$\approx 15.7$$

$$AM = 15.7 \text{ cm [3]}$$

(c) Calculate the shortest distance from A to BC .

$$\sin 54 = \frac{AD}{17.2}$$

$$AD = 13.91$$

$$\approx 13.9$$

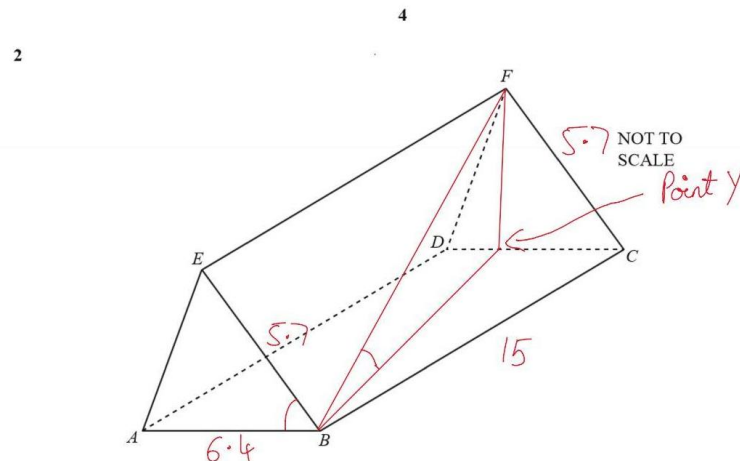
$$13.9 \text{ cm [3]}$$

Q6a - Using the big triangle ABC as a whole, 2 angles a side length are known. AC can be found by applying the sine law.

Q6b - Using triangle AMC , 2 side lengths and the angle in between them are known. Cosine law can be applied to find the remaining side AM .



Q6c - Shortest distance from a point to a line is always perpendicular to the line (a 90 degree angle is being made). The point at which the red line touches line BC can be named D. Using triangle ADB, the hypotenuse and an angle are known so the remaining side, AD, can be found.



The diagram shows a solid triangular prism $ABCDEF$ of length 15 cm. $AB = 6.4$ cm, $EB = 5.7$ cm and the volume of the prism is 145 cm^3 .

- (a) Show that angle $EBA = 32^\circ$, correct to the nearest degree.

$$\begin{array}{l} \text{Area } ABE = \frac{1}{2} (6.4)(5.7) \sin EBA \\ = 18.24 \sin EBA \end{array} \quad \left| \begin{array}{l} 18.24 \sin EBA \times 15 = 145 \\ 273.6 \times \sin EBA = 145 \\ \sin EBA = 0.5299 \\ EBA = 32.00 \\ \approx 32^\circ \end{array} \right. \quad [3]$$

- (b) Find the length of EA .

$$\begin{array}{l} EA^2 = 5.7^2 + 6.4^2 - 2(6.4)(5.7) \cos 32 \\ EA^2 = 73.45 - 61.87 \\ EA^2 = 11.57 \\ EA = 3.4 \end{array}$$

3.4 cm [3]



5

(c) Calculate the shortest distance from E to AB.

Handwritten solution:

$$\sin 32 = \frac{x}{5.7}$$

$$x = 3.02$$

..... 3.02 cm [3]

(d) Calculate the angle BF makes with the base, ABCD, of the prism.

Handwritten solution:

$$BF^2 = 5.7^2 + 15^2$$

$$BF = 16.05$$

$$\sin Q = \frac{3.02}{16.05}$$

$$Q = 10.8$$

..... 10.8 [4]

(e) The prism is made of plastic with density 938 kg/m^3 .

Calculate the mass of the prism in grams.
[Density = mass ÷ volume]

Handwritten solution:

$$938 = \frac{\text{mass}}{\text{volume}}$$

$$\frac{145 \text{ cm}^3}{100^3} = 1.45 \times 10^{-4} \text{ m}^3$$

$$938 \times 1.45 \times 10^{-4} = \text{mass}$$

$$m = 0.13601 \text{ Kg}$$

$$0.13601 \times 1000 = 136.01 \text{ grams} \approx 136$$

..... 136 g [3]

Q2a - Using the vertical triangle ABE, 2 side lengths and its area are known. The angle between the lengths needs to be proven. Use the formula for the area of a triangle to prove the angle.

Q2b - In the triangle ABE, 2 side lengths and the angle opposite to the unknown side is known. The cosine rule can be applied to find EA.

Q2c - Shortest distance is the perpendicular distance. Another right angled triangle is formed from which the required length can be found.

Q2d - Another triangle (FYB) needs to be formed for this question. Assume point Y is such that FY is the shortest distance from point F to line DC. This

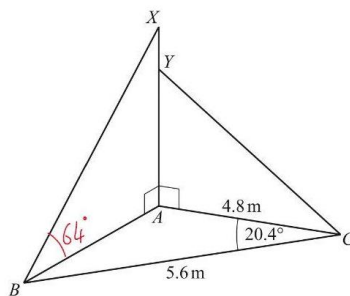


shortest distance will be the same as the shortest distance calculated in the previous part because the triangles on both sides of the prism are identical. Triangle FYB can be assumed to be inside the prism for the purposes of understanding. The rectangle BCFE can be cut in half from its diagonal, BF, to give another triangle from which the length of BF can be found. This length can then be used in triangle FYB to find the required angle.

Q2e - The main concept being tested in this question is unit conversions. The density is given in kg/m^3 whereas the volume of the prism given in the question is given in cm^3 . First of all, cm^3 needs to be converted into m^3 . To convert a quantity from cm to m , you need to divide it by 100. If a volume needs to be converted, the quantity must be divided by 100^3 . The volume in m^3 is now known. Rearrange the given formula for density to find mass in kilograms. Multiply this by 1000 to give the mass in grams, which is the final answer.



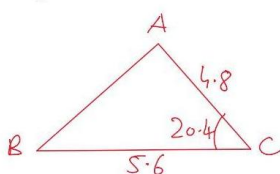
5 (a)



NOT TO SCALE

ABC is a scalene triangle on horizontal ground.
 AYX is a straight vertical post, held in place by two straight wires XB and YC .
 $AC = 4.8$ m, $BC = 5.6$ m and angle $ACB = 20.4^\circ$.

(i) Calculate AB .

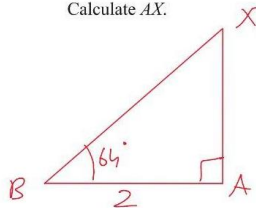


$$\begin{aligned} AB^2 &= 5.6^2 + 4.8^2 - 2(5.6)(4.8)\cos 20.4 \\ AB^2 &= 54.4 - 50.38 \\ AB^2 &= 4.01 \\ AB &= 2.00 \end{aligned}$$

$AB = 2$ m [3]

(ii) Angle $XBA = 64^\circ$.

Calculate AX .

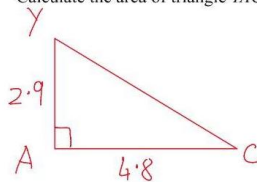


$$\begin{aligned} \tan 64 &= \frac{AX}{2} \\ AX &= 4.1 \end{aligned}$$

$AX = 4.1$ m [2]

(iii) $AY = 2.9$ m.

Calculate the area of triangle YAC .

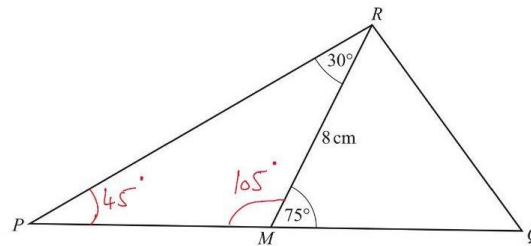


$$\begin{aligned} \frac{1}{2}(2.9)(4.8) \\ = 6.96 \end{aligned}$$

6.96 m² [2]



(b)



NOT TO
SCALE

In triangle PQR , M is the midpoint of PQ .
 $RM = 8$ cm, angle $PRM = 30^\circ$ and angle $RMQ = 75^\circ$.

Calculate PQ .

$$\frac{\sin 45^\circ}{8} = \frac{\sin 30^\circ}{PM}$$

$$PM \sin 45^\circ = 4$$

$$PM = 4\sqrt{2}$$

$$PQ = 2PM$$

$$\begin{aligned} PQ &= 8\sqrt{2} \\ &= 11.313 \\ &\approx 11.3 \end{aligned}$$

$$PQ = \dots\dots\dots 11.3 \dots\dots\dots \text{cm [5]}$$

Q5a(i) - Cosine law will be applied because 2 sides and the angle in between them is known.

Q5a(ii) - The vertical triangle BAX will now be used. A side and an angle is known, the other side can easily be found by using tan.

Q5a(iii) - The other triangle, YAC, is now being used. 2 lengths, on either side of the right angle, are known and the area is asked for. Plug in the values of the lengths into the formula for the area of a right angle triangle to get the final answer.



Q5b - Angle RMQ is known so angle RMP can also be calculated because angles on a straight line add up to 180 degrees. Angles in a triangle also add up to 180 degrees so angle RPM can also be calculated. The sine rule can be used to find PM. Since M is the midpoint of line PQ, PM is equal to MQ. $PM + MQ = PQ$. The length of PM can be doubled to find PQ which is the quantity required by the question.

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UNIT 7: Vectors and Transformation

7 Transformations and vectors

E7.1 Transformations	Notes and examples
Recognise, describe and draw the following transformations: 1 Reflection of a shape in a straight line. 2 Rotation of a shape about a centre through multiples of 90°. 3 Enlargement of a shape from a centre by a scale factor. 4 Translation of a shape by a vector $\begin{pmatrix} x \\ y \end{pmatrix}$.	Questions may involve combinations of transformations. A ruler must be used for all straight edges. Positive, fractional and negative scale factors may be used.

E7.2 Vectors in two dimensions	Notes and examples
1 Describe a translation using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or a. 2 Add and subtract vectors. 3 Multiply a vector by a scalar.	Vectors will be printed as $\overrightarrow{AB}$ or a.

E7.3 Magnitude of a vector	Notes and examples
Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$.	The magnitudes of vectors will be denoted by modulus signs, e.g. • $\mathbf{a}$ is the magnitude of a • $\overrightarrow{AB}$ is the magnitude of $\overrightarrow{AB}$.

E7.4 Vector geometry	Notes and examples
1 Represent vectors by directed line segments. 2 Use position vectors. 3 Use the sum and difference of two or more vectors to express given vectors in terms of two coplanar vectors. 4 Use vectors to reason and to solve geometric problems.	Examples include: • show that vectors are parallel • show that 3 points are collinear • solve vector problems involving ratio and similarity.

Transformation

There are 4 types of transformation

Reflection:

An isometric transformation

Line of reflection: An imaginary line across which a shape is reflected to create its mirror image.

How can we reflect a shape?



STEP 1:

From a point on the original object measure the perpendicular distance to the mirror line

STEP 2:

Continuing from that point on the mirror line, and in the same direction, measure the same distance again

STEP 3:

Mark the corresponding point on the reflected image at the position you have reached

STEP 4:

Join together the reflected points and label the reflected image

1. Vertical lines (of the form $x=k$, for some number k)

The perpendicular distance can be found by counting the number of squares horizontally from a point on the original object until you reach the mirror line

2. Horizontal lines (of the form $y=k$, for some number k)

The perpendicular distance can be found by counting the number of squares vertically from a point on the original object until you reach the mirror line

3. Diagonal lines (of the form $y=mx+c$, see straight line graphs)

Reflecting in diagonal lines can be much trickier

The perpendicular distance can be measured directly using a ruler

You can also count the number of squares horizontally and vertically until you reach the mirror line, e.g. 2 to the right, 3 down, then from the same point on the mirror line, count the number of squares vertically that was the previous horizontal distance and count the number of squares horizontally that was the previous vertical distance, e.g. 3 to the right, 2 down

4. Double Reflections

Double reflections are where the mirror line passes through the shape being reflected

Part of the shape gets reflected on one side of the mirror line, the other part gets reflected on the other side

How can I describe reflection?

State that the transformation is a reflection

Give the mathematical equation of the mirror line

Rotation:

Isometric

How can we rotate the shape?

The easiest way to draw a rotation is to use tracing paper, this should be available to you in an exam but you may have to ask an invigilator for it



STEP 1:

Place the tracing paper over page and draw over the original object

STEP 2:

Place the point of your pencil on the centre of rotation

STEP 3:

Rotate the tracing paper the angle that has been asked for in the question, it will usually be an "easy" angle such as 90° , 180° or 270°

STEP 4:

Your tracing paper is in the position showing you where to draw the rotated image, carefully draw the image onto the coordinate grid

How can we describe rotation?

State that the transformation is a rotation

State the centre of rotation (the point about which the object is rotated)

State the angle of rotation (how many degrees around the point that the object has been rotated)

State the direction of rotation (clockwise or anticlockwise, unless the angle is 180° , then a direction is not required)

Enlargement:

An enlargement is a transformation that changes the size of the shape

The scale factor tells you how many times bigger each edge of the enlarged image will be compared to the corresponding edge on the original object

If the scale factor is greater than 1, the enlarged image will be bigger than the original object

If the scale factor is less than 1, the enlarged image will be smaller than the original object

The position of a shape will also change with enlargement

The orientation of the shape will be the same for a positive enlargement

How can enlarge a shape?

You need to be able to perform an enlargement (on a coordinate grid)

STEP 1:

Starting from the centre of enlargement, count the horizontal and vertical distances to

one vertex on the original object

STEP 2:

Multiply the distances by the given scale factor

STEP 3:

Starting again from the centre of enlargement, measure the new distances and mark the position on the grid of the corresponding vertex on the enlarged image



The distances from the centre of enlargement to the enlarged image will be in the same direction for a positive scale factor and the opposite direction for a negative scale factor

STEP 4:

Repeat STEPS 1 to 3 for the remaining vertices

STEP 5:

Connect the vertices on the enlarged image and label it

How can we describe an enlargement?

You need to be able to identify and describe an enlargement when presented with one

You must fully describe a transformation to get full marks

For an enlargement, you must:

State that the transformation is an enlargement

State the scale factor

Give the coordinates of the centre of enlargement

Translation:

What is a translation?

A translation is the movement of a shape

The size, shape and orientation (which way up it is) of the shape stays the same

How can we translate a shape?

The movement of a translation is described by a vector

You need to know how to write a translation using a vector (rather than words)

Vectors are written as column vectors in the form $\begin{pmatrix} X \\ Y \end{pmatrix}$ where:

X is the distance moved horizontally

Negative means move to the left

Positive means move to the right

Y is the distance moved vertically

Negative means move down

Positive means move up

STEP 1:

Select a vertex on the original object, move it according to the information given in the column vector

STEP 2:

Repeat STEP 1 for each of the vertices on the original object



STEP 3:

Connect the translated vertices and label the translated image

In some cases, where the vectors are small enough, the image can overlap the object

The vector is how the shape moves not the size of the gap between the object and the image, watch out for this common error!

How can we describe a translation?

It is important to fully describe a transformation in order to get full marks

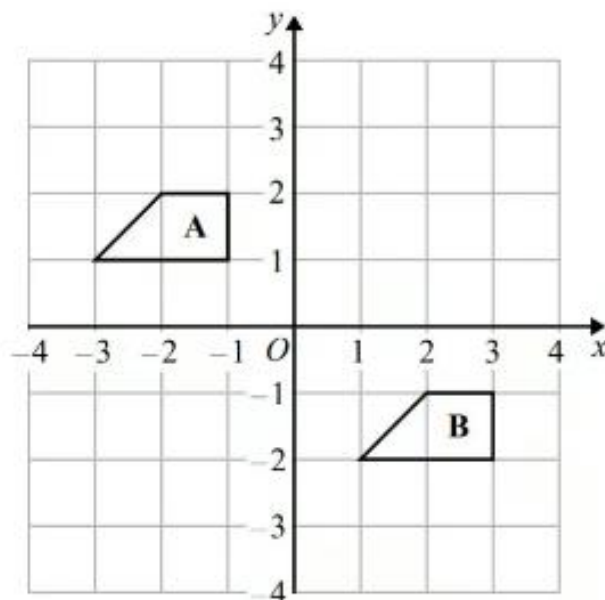
For a translation, you must:

State that the transformation is a translation

Give the column vector that describes the movement

Solved Example:

Q1)

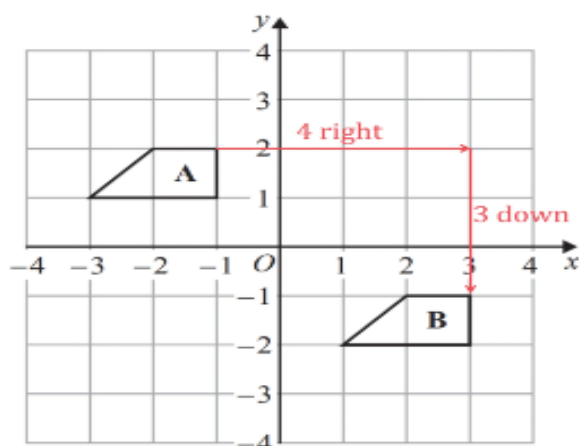


Describe the single transformation that maps shape **A** onto shape **B**.



The shape has been moved so this is a translation.

Pick a point on the original shape and find its corresponding point on the translated shape.
Count how many units it has moved horizontally and vertically.



Write as a vector. The horizontal displacement goes at the top (right is positive, left is negative).
The vertical displacement goes at the bottom (up is positive, down is negative).

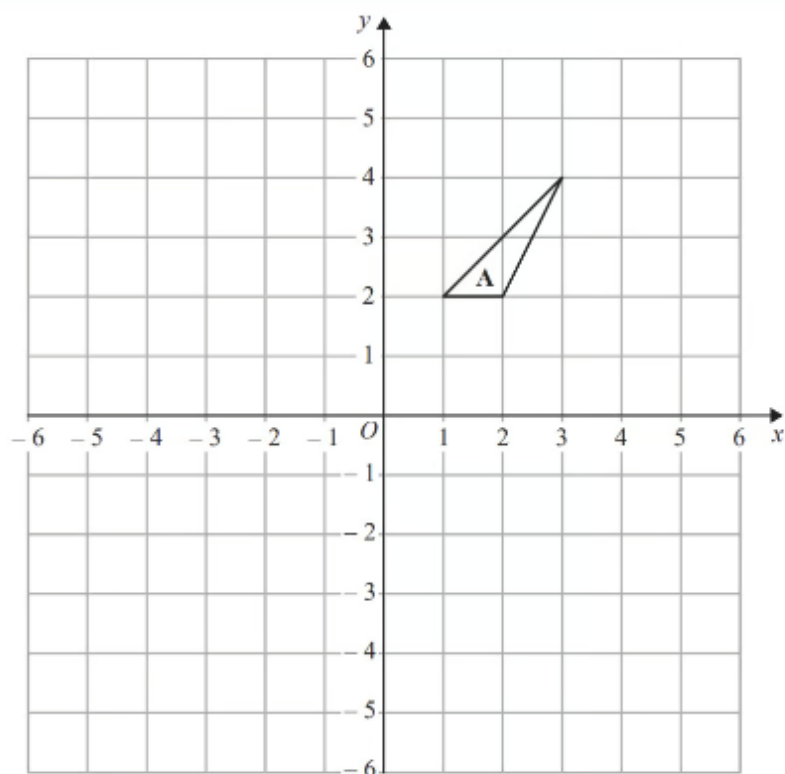
Translation by $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Writing "Translation" [1]

Correct vector [1]



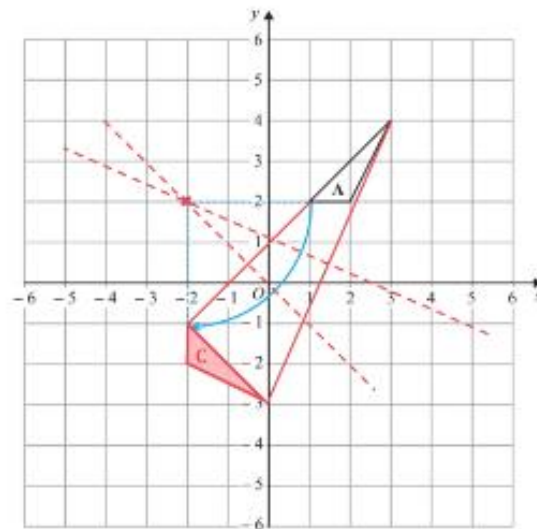
Q2



Triangle **A** is rotated 90° clockwise about the point $(0, 1)$ to give triangle **B**.

Triangle **B** is translated by the vector $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ to give triangle **C**.

Describe fully the single transformation that maps triangle **A** onto triangle **C**.



You could also use trial and error with tracing paper. Start with trying the centre as the origin and work outwards if needed.

Rotation 90° clockwise about the centre $(-2, 2)$

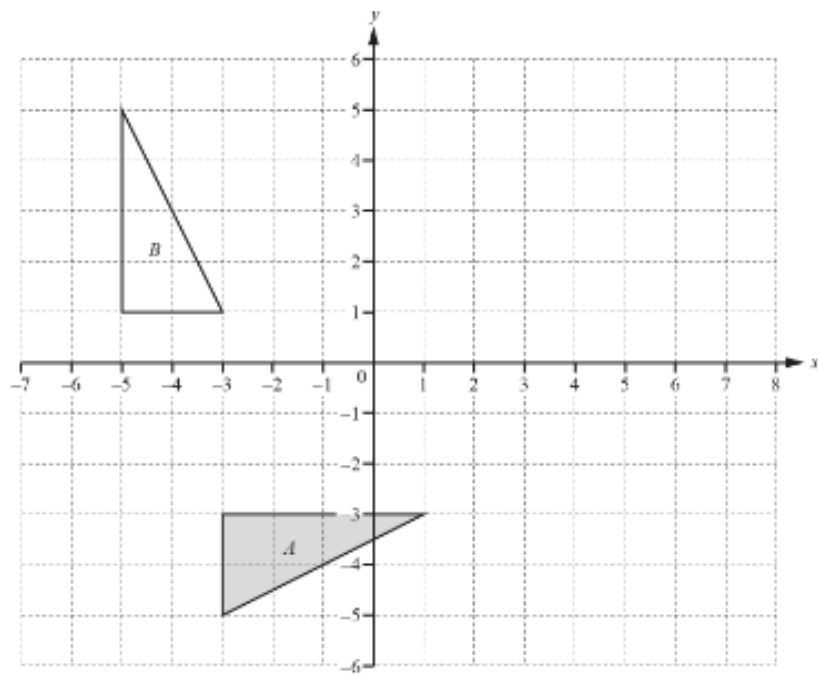
Writing "Rotation" with correct angle [1]

Writing "Rotation" with correct centre [1]

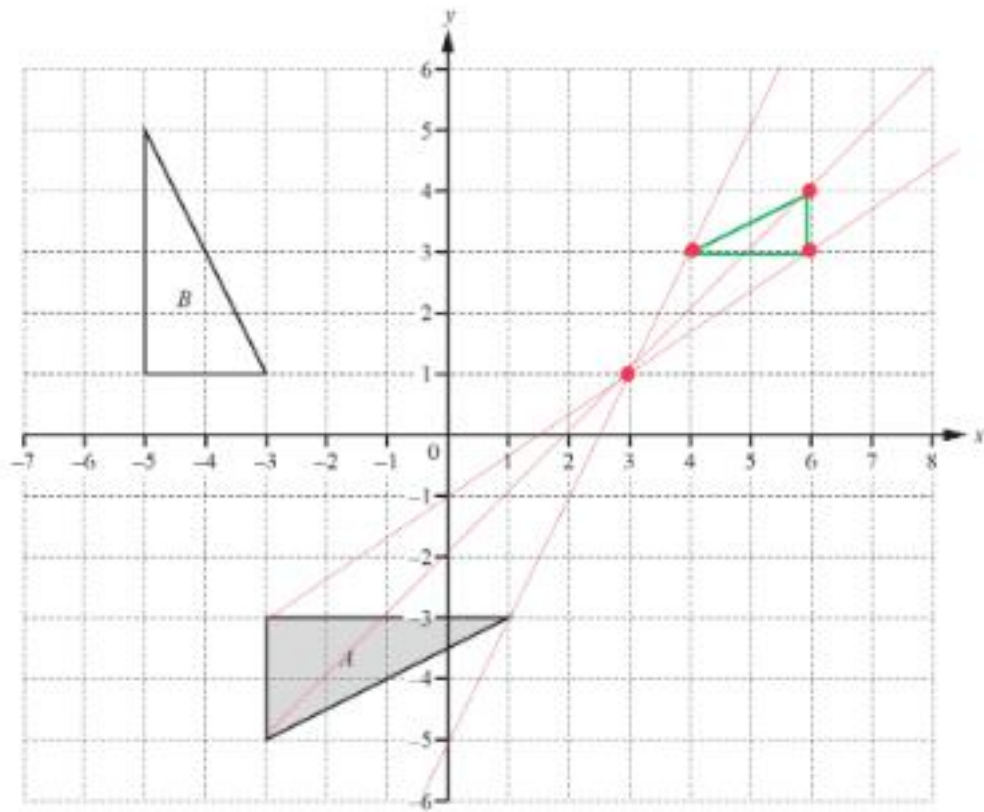
It is acceptable to write: 270° anticlockwise



Q3)



Draw the image of triangle A after an enlargement by scale factor $-\frac{1}{2}$, centre $(3, 1)$.





Vectors

What are vectors?

A vector is a type of number that has both a size and a direction

Here we only deal with two-dimensional vectors, although it is possible to have vectors with any number of dimensions

Representing vectors

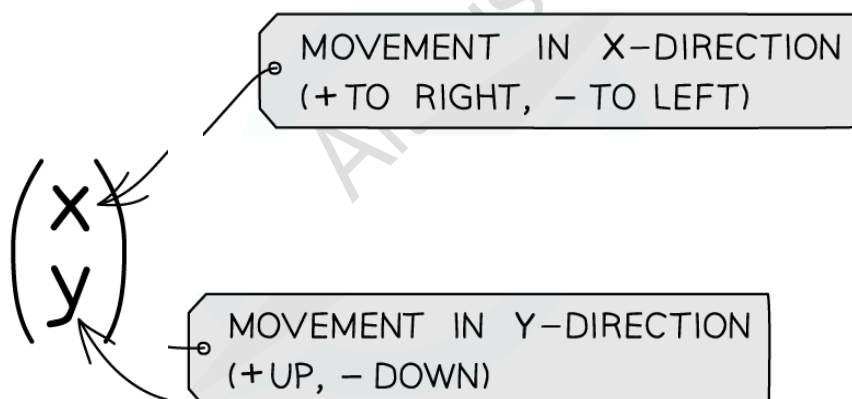
Vectors are represented as arrows, with the arrowhead indicating the direction of the vector, and the length of the arrow indicating the vector's magnitude (ie its size)

In print vectors are usually represented by bold letters (as with vector **a** in the diagram above), although in handwritten workings underlined letters are normally used, a.

Another way to indicate a vector is to write its starting and ending points with an arrow symbol over the top.

Vectors and transformation geometry

In transformation geometry, translations are indicated in the form of a column vector:



Vectors on a grid

You also need to be able to work with vectors on their own, outside of the transformation geometry context

When vectors are drawn on a grid (with or without x and y axes), the vectors can be represented in the same $\begin{pmatrix} x \\ y \end{pmatrix}$ column vector form as above

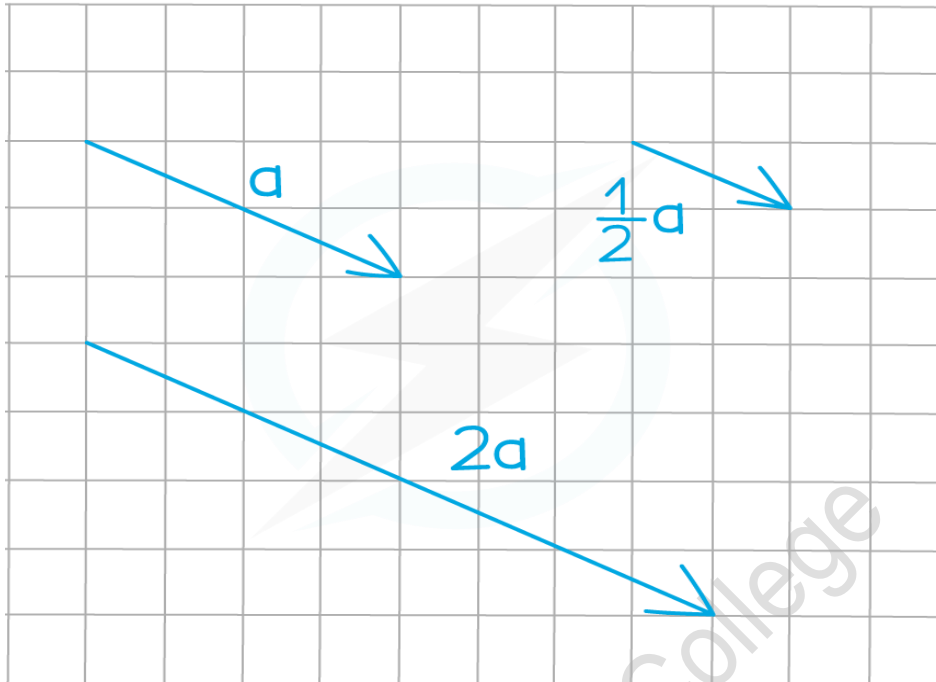
Multiplying a vector by a scalar

A scalar is a number with a magnitude but no direction – ie the regular numbers you are used to using

When a vector is multiplied by a positive scalar, the magnitude of the vector changes, but its direction stays the same



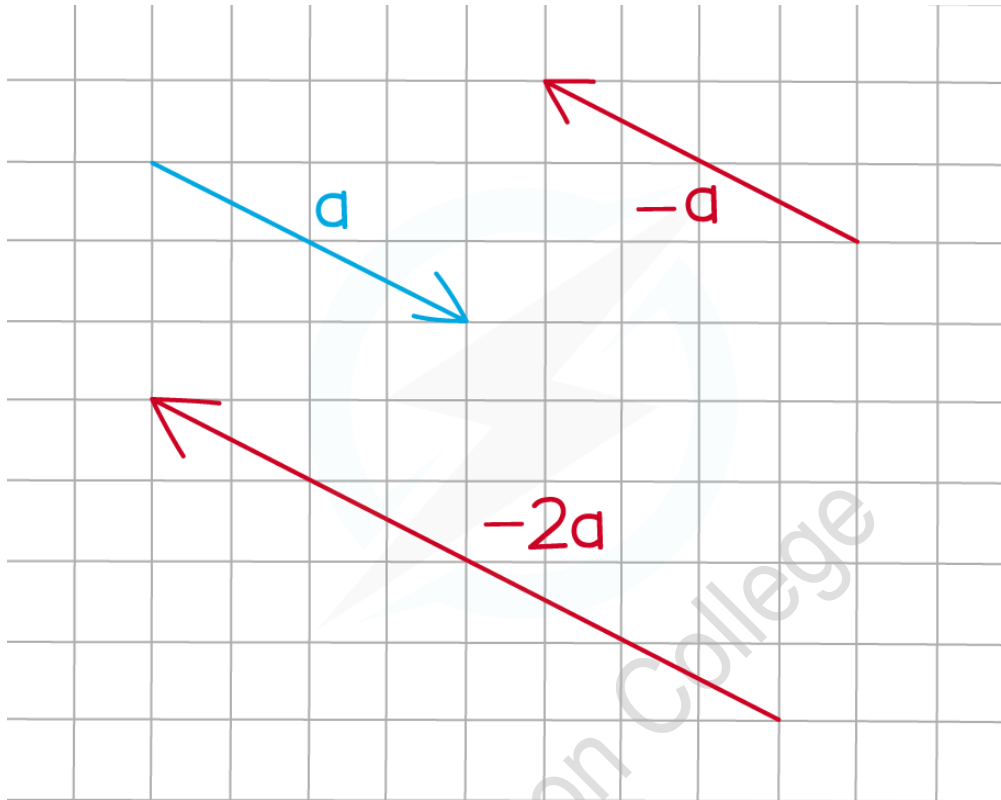
If the vector is represented as a column vector, then each of the numbers in the column vector gets multiplied by the scalar



$$\mathbf{a} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}, \quad 2\mathbf{a} = \begin{pmatrix} 2 \times 4 \\ 2 \times -2 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}, \quad \frac{1}{2}\mathbf{a} = \begin{pmatrix} 0.5 \times 4 \\ 0.5 \times -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$



Note that multiplying by a negative scalar also changes the direction of the vector:



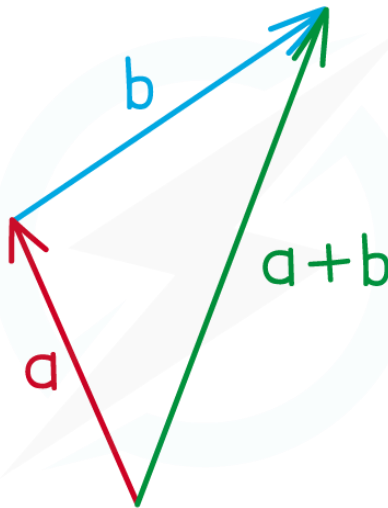
$$\mathbf{a} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}, \quad -\mathbf{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}, \quad -2\mathbf{a} = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$$

Note in particular that vector $-\mathbf{a}$ is the the same size as vector $\mathbf{a}$, but points in the opposite direction!

Adding and subtracting vectors



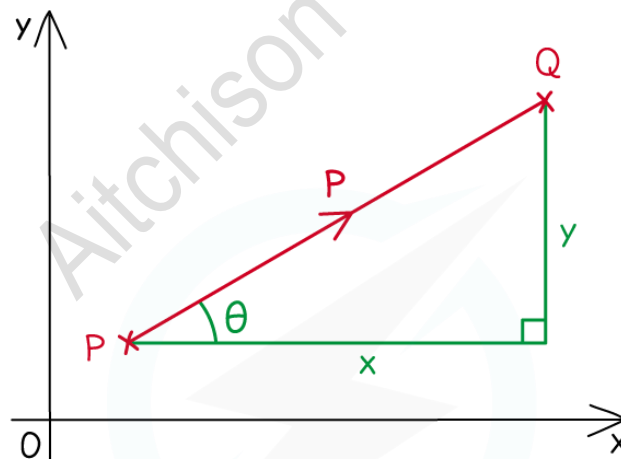
Adding two vectors is defined geometrically, like this:



Subtracting one vector from another is thought of as adding a negative vector
 $a - b = a + (-b)$

When vectors are represented as column vectors, adding or subtracting is simply a matter of adding or subtracting the vectors' x and y coordinates

What is the magnitude or modulus of a vector?



THE DISTANCE PQ IS CALLED THE
MAGNITUDE (OR MODULUS) OF
THE VECTOR P

THIS IS DENOTED BY $|P|$ OR $|\vec{PQ}|$

This depends on the use of the vector

For velocity, magnitude would be speed

For a force, magnitude would be the strength of the force (in Newtons)



The words magnitude and modulus mean the same thing with vectors

In geometry magnitude and modulus mean the distance of the vector

This is always a positive value

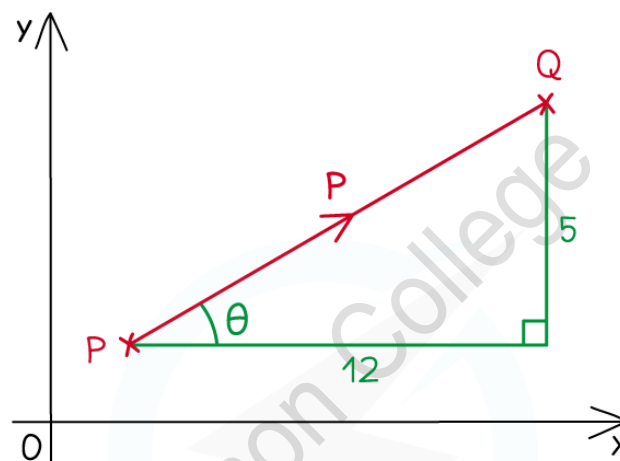
The direction of the vector is irrelevant

Magnitude or modulus is indicated by vertical lines

$|a|$ would mean the magnitude of vector a

How do I find the magnitude or modulus of a vector

Pythagoras' Theorem!



$$P = \vec{PQ} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

$$|P| = |\vec{PQ}| = \sqrt{12^2 + 5^2} = 13$$

IF THE COORDINATES OF P AND Q
ARE KNOWN THE "12" AND "5" CAN
BE CALCULATED

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Vector Problem Solving

What are vector proofs?

In vector proofs we use vectors, along with a few key ideas, to prove that things are true in geometrical diagrams



Problem solving with vectors involves using these vector proofs to help us to find out additional information

Parallel vectors

Two vectors are parallel if and only if one is a multiple of the other

This tends to appear in vector proofs in the following ways:

If you find in your workings that one vector is a multiple of the other, then you know that the two vectors are parallel – you can then use that fact in the rest of the proof

If you need to *show* that two vectors are parallel, then all you need to do is show that one of the vectors multiplied by some number is equal to the other one

Points on a straight line

Often you are asked to show in a vector proof that three points lie on a straight line (ie that they are collinear)

To show that three points a, b and c lie on a straight line,

show that the vectors connecting the three points are parallel,

for example, show that AB is a multiple of (and therefore parallel to) AC, or that AB is a multiple of (and therefore parallel) to BC

as those two vectors are parallel and they share a common point it means that the three lines form a straight line

Vectors divided in ratios

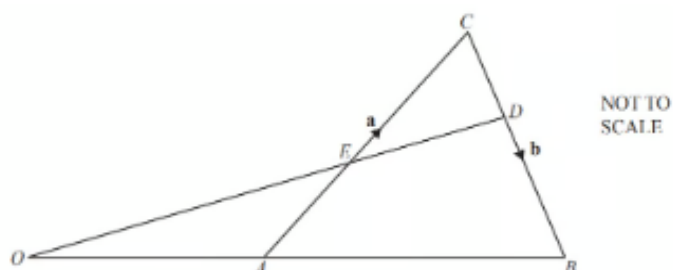
Be careful turning ratios into fractions in vector proofs!

If a point divides a line segment in the ratio $p : q$, then:

$$\overrightarrow{AX} = \frac{p}{p+q} \overrightarrow{AB} \text{ and } \overrightarrow{XB} = \frac{q}{p+q} \overrightarrow{AB}$$



Solved Questions:



In the diagram, OAB and OED are straight lines.

O is the origin, A is the midpoint of OB and E is the midpoint of AC .

$\overrightarrow{AC} = \mathbf{a}$ and $\overrightarrow{CB} = \mathbf{b}$.

Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, in its simplest form

i) $\overrightarrow{AB}$,

$$\overrightarrow{AB} = \dots\dots\dots [1]$$

ii) $\overrightarrow{OE}$,

$$\overrightarrow{OE} = \dots\dots\dots [2]$$

iii) the position vector of D .

[3]



- i) Identify the path from A to B .

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AC} + \overrightarrow{CB} \\ &= \mathbf{a} + \mathbf{b}\end{aligned}$$

$$\overrightarrow{AB} = \underline{\mathbf{a}} + \underline{\mathbf{b}} \quad [1]$$

- ii) Identify the path from O to E .

$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$$

[1]

A is the midpoint of AB so $\overrightarrow{OA} = \overrightarrow{AB} = \mathbf{a} + \mathbf{b}$.

E is the midpoint of AC so $\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}\mathbf{a}$.

Substitute these into the route for the path.

$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE} = (\mathbf{a} + \mathbf{b}) + \frac{1}{2}\mathbf{a}$$

Simplify.

$$\overrightarrow{OE} = \underline{\frac{3}{2}\mathbf{a}} + \underline{\mathbf{b}} \quad [1]$$

- iii) The position vector of D is OD .

Identify a path from O to D that uses some of the paths you already know vectors for.

$$\overrightarrow{OD} = \overrightarrow{OE} + \overrightarrow{ED}$$

E is the midpoint of AC so $\overrightarrow{EC} = \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}\mathbf{a}$. Therefore $\overrightarrow{ED}$ must be equal to $\frac{1}{2}\mathbf{a} + x\mathbf{b}$

where x is a constant to be found.

$\overrightarrow{ED}$ is parallel to $\overrightarrow{OE}$ so $\overrightarrow{ED} = k(\overrightarrow{OE}) = k\left(\frac{3}{2}\mathbf{a} + \mathbf{b}\right)$.

$$\frac{1}{2}\mathbf{a} + x\mathbf{b} = k\left(\frac{3}{2}\mathbf{a} + \mathbf{b}\right)$$

Compare coefficients of $\mathbf{a}$.

$$\frac{1}{2}\mathbf{a} = k\left(\frac{3}{2}\mathbf{a}\right)$$

$$\frac{3}{2}k = \frac{1}{2}$$



Compare coefficients of **a**.

$$\frac{1}{2}\mathbf{a} = k\left(\frac{3}{2}\mathbf{a}\right)$$

$$\frac{3}{2}k = \frac{1}{2}$$

$$k = \frac{1}{3}$$

Compare coefficients of **b**.

$$x\mathbf{b} = k\mathbf{b} = \frac{1}{3}\mathbf{b} \text{ so } \overrightarrow{ED} = \frac{1}{2}\mathbf{a} + \frac{1}{3}\mathbf{b}.$$

[1]

Substitute these into the route for the path.

$$\overrightarrow{OD} = \overrightarrow{OE} + \overrightarrow{ED} = \left(\frac{3}{2}\mathbf{a} + \mathbf{b}\right) + \left(\frac{1}{2}\mathbf{a} + \frac{1}{3}\mathbf{b}\right)$$

[1]

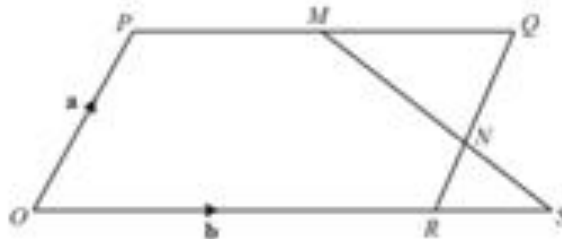
Simplify.

$$\overrightarrow{OD} = \left(\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{a}\right) + \left(\mathbf{b} + \frac{1}{3}\mathbf{b}\right)$$

$$\overrightarrow{OD} = 2\mathbf{a} + \frac{4}{3}\mathbf{b} \quad [1]$$



24



NOT TO
SCALE

O is the origin and $OPQR$ is a parallelogram.
 M is the midpoint of PQ and N divides QR in the ratio $2 : 1$.
 $\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{OR} = \mathbf{b}$.

- (a) Find $\overrightarrow{MN}$.
 Give your answer in terms of $\mathbf{a}$ and/or $\mathbf{b}$ and in its simplest form.

$$\overrightarrow{MN} = \text{_____} [2]$$

- (b) The lines MN and OR are extended to meet at S .

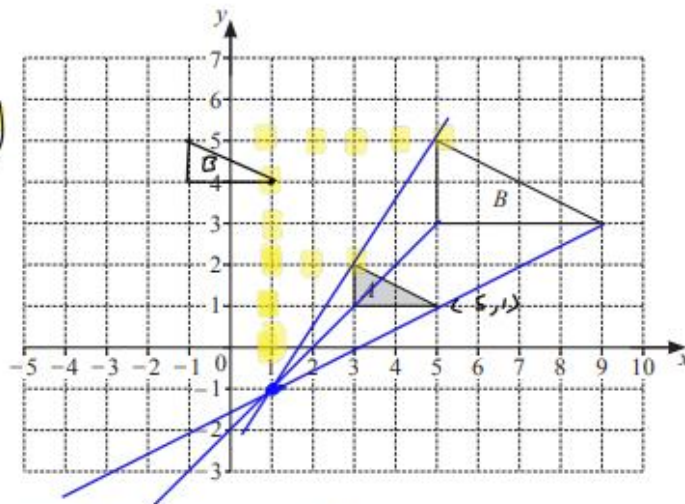
Find the position vector of S .
 Give your answer in terms of $\mathbf{a}$ and/or $\mathbf{b}$ and in its simplest form.

$$\text{_____} [3]$$



ANSWER KEY:

9
Trans
(5, 1) $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$
(1, 4)



- (a) Describe fully the **single** transformation that maps triangle *A* onto triangle *B*.

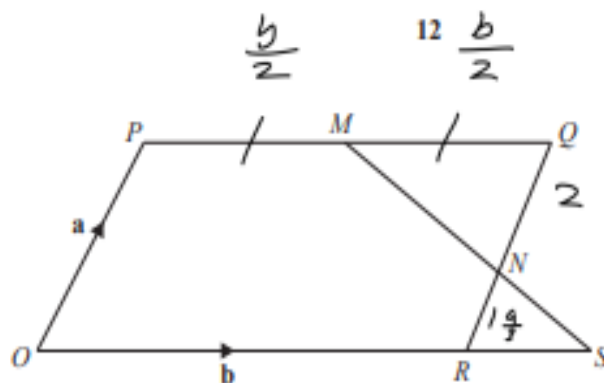
Enlargement, Centre of Enlargement = (1, -1)
Scale factor = +2 [3]

- (b) On the grid, draw the image of triangle *A* after a translation by the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$. 13 [2]

10 Write 174 000 in standard form.



24



⑥ NOT TO SCALE
 $3 \vec{RN} = 1 \vec{RS}$
 $\vec{RN} = \frac{1}{3} (\vec{a})$

O is the origin and OPQR is a parallelogram.

Q and

$\vec{OP} = \vec{a}$ and $\vec{OR} = \vec{b}$.

$\vec{MN} = \vec{MQ} + \vec{QN} + \vec{NR} + \vec{RN}$
 $= (-\frac{b}{2}) + (-\vec{a}) + \vec{b} + (\frac{a}{3})$

(a) Find $\vec{MN}$.

Give your answer in terms of $\vec{a}$ and/or $\vec{b}$ and in its simplest form.

$\vec{PM} = \vec{MQ}$ { $\vec{PM} + \vec{MQ} = \vec{b}$ so, $\vec{PM} = \frac{b}{2}$ }
 $\vec{RN} = \vec{a}$ { $\vec{RN} = \frac{a}{3}$ }

$= -\vec{a} + \frac{a}{3} + \vec{b} - \frac{b}{2}$

$= \frac{-3\vec{a} + \vec{a}}{3} + \frac{2\vec{b} - \vec{b}}{2}$
 $= \frac{-2\vec{a}}{3} + \frac{\vec{b}}{2}$

$\vec{MN} = \frac{-2\vec{a}}{3} + \frac{\vec{b}}{2}$ [2]

(b) The lines MN and OR are extended to meet at S.

Find the position vector of S.

Give your answer in terms of $\vec{a}$ and/or $\vec{b}$ and in its simplest form.

$\triangle MAN$ is similar to $\triangle SRN$

Because: ① $\angle MAN = \angle SRN$ (V.O.P.P) ② $\angle MNA = \angle SRN$ (alt $\angle$'s)

so

$\frac{AN}{RN} = \frac{MN}{SN} = \frac{MR}{SR}$

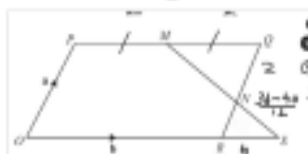
$\frac{2}{1} = \frac{1}{1} = \frac{1}{1}$

$SR : MA = 1 : 2$

$2 \vec{SR} = 1 \vec{MA}$

$\vec{SR} = \frac{1}{2} (\frac{b}{2})$

$\vec{SR} = \frac{b}{4}$



$\vec{MN} : \vec{SN} = 2 : 1$

so $2 \vec{SN} = 1 \vec{MN}$

$2 \vec{SN} = \frac{1}{2} (\frac{b}{2} - \frac{2a}{3})$

$\vec{SN} = \frac{1}{4} (\frac{3b - 4a}{6})$

$\vec{SN} = \frac{3b - 4a}{24}$

$\vec{OS} = \vec{b} + \frac{b}{4}$

$= \frac{4b + b}{4}$

$= \frac{5b}{4}$

$\vec{OS} = \frac{5b}{4}$ [3]



Unit 8 : Probability

8 Probability

E8.1 Introduction to probability	Notes and examples
1 Understand and use the probability scale from 0 to 1.	$P(A)$ is the probability of A $P(A')$ is the probability of not A
2 Understand and use probability notation.	
3 Calculate the probability of a single event.	Probabilities should be given as a fraction, decimal or percentage. Problems may require using information from tables, graphs or Venn diagrams.
4 Understand that the probability of an event not occurring = $1 -$ the probability of the event occurring.	e.g. $P(B) = 0.8$, find $P(B')$
E8.2 Relative and expected frequencies	Notes and examples
1 Understand relative frequency as an estimate of probability.	e.g. use results of experiments with a spinner to estimate the probability of a given outcome.
2 Calculate expected frequencies.	e.g. use probability to estimate an expected value from a population. Includes understanding what is meant by fair, bias and random.
E8.3 Probability of combined events	Notes and examples
Calculate the probability of combined events using, where appropriate: - sample space diagrams - Venn diagrams - tree diagrams.	Combined events could be with or without replacement. The notation $P(A \cap B)$ and $P(A \cup B)$ may be used in the context of Venn diagrams. On tree diagrams outcomes will be written at the end of branches and probabilities by the side of the branches.
E8.4 Conditional probability	Notes and examples
Calculate conditional probability using Venn diagrams, tree diagrams and tables.	Knowledge of notation, $P(A B)$, and formulas relating to conditional probability is not required.

IGCSE Probability

C8.1 Introduction to Probability

1. Understanding the Probability Scale

Probability Scale: Ranges from 0 to 1.

0: The event cannot happen (impossible event).

1: The event is certain to happen (certain event).

Values between 0 and 1: Represent the likelihood of the event occurring.



2. Calculating the Probability of a Single Event

Formula:

Probability Formulas

$$P(A) = \frac{\text{Numbers of Favourable Outcomes}}{\text{Total Numbers of Favourable Outcomes}}$$

Example: If there are 3 red counters and 7 blue counters in a bag, the probability of picking a red counter is:

$$P(\text{Red}) = 3/10 = 0.3 = 30\%$$

3. Probability of an Event Not Occurring

Complement Rule:

$$P(\text{Not } A) = 1 - P(A)$$

Example:

If the probability that a counter is blue is $P(\text{Blue}) = 0.8$ then:

$$P(\text{Not Blue}) = 1 - 0.8 = 0.2 (\text{or } 20\%)$$

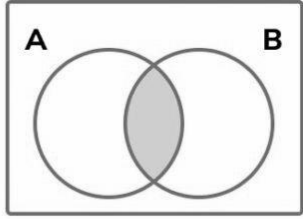
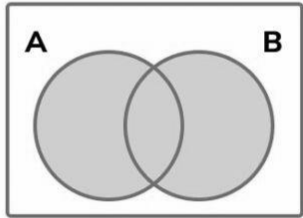
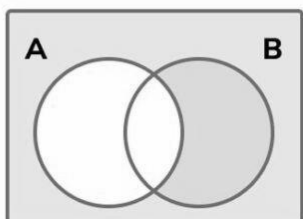
4. Using Information from Tables, Graphs, and Venn Diagrams

Tables: You can use tables to determine probabilities based on given data.

Graphs: Probability can also be visualized in graphical representations like bar charts.

Venn Diagrams: Used for two sets, showing the relationship between different probabilities.



$A \cap B$	'A and B' The intersection of A and B. The elements in both sets A and B.	
$A \cup B$	'A or B' The union of A or B. Any element in set A or set B.	
A'	'Not A' The complement of A. Any element not in A.	

Summary

Probability Values: Can be expressed as fractions, decimals, or percentages.

Key Concepts: Single event probabilities, the complement of an event, and the use of data representations to derive probabilities.

(make sure to revise the signs for intersection, union, complement and do not mix them up as it is one of the most common mistakes made in the exams)

C8.2 Relative and Expected Frequencies

1. Understanding Relative Frequency as an Estimate of Probability

Relative frequency is the number of times an event happens divided by the total number of outcomes that took place in an experiment, known as the number of trials.

To calculate the relative frequency we can use the formula

$$\text{Relative frequency} = \frac{\text{frequency of the event occurring}}{\text{total number of trials of the experiment}}$$

Example:

If you spin a spinner 50 times and it lands on red 15 times:

Relative Frequency of Red = $15/50 = 0.3$ (or 30%)



Interpretation: The relative frequency approximates the probability of landing on red in a larger number of spins.

(learn to interpret these questions before trying to solve them, as you cannot solve a question you don't fully understand)

2. Calculating Expected Frequencies

Expected frequency is the number of times we would expect an event to occur over a given number of trials that take place during an experiment.

$$\text{Expected frequency} = \text{Probability of event} \times \text{number of trials}$$

 **Example** The probability of a spinner landing on a 2 is 0.3

If the spinner is spun 500 times,
the expected frequency of getting a 2 is $0.3 \times 500 = 150$

We would **expect** to see 150 2s in 500 spins.

Note that this might not be the case in reality!

Expected Frequency: The predicted number of times an event will occur in a certain number of trials based on its probability.

Formula:

$$\text{relative frequency} = \frac{\text{frequency}}{\text{number of trials}}$$

Example:

If the probability of landing on blue in a spinner is **P(Blue) = 0.4** and you spin the spinner 100 times:

Expected Frequency of Blue = $100 \times 0.4 = 40$

This means you expect to land on blue approximately 40 times.

3. Understanding Key Concepts

Fair: An experiment or device is fair if all outcomes have an equal probability.

Example: A fair six-sided die has a $1/6$ chance for each number.

Bias: When an experiment or device consistently favors certain outcomes over others, leading to inaccurate probabilities.

Example: A biased die that has a higher chance of landing on 6.

Random: Events that occur without any predictable pattern or influence, ensuring that each outcome has a fair chance.

Example: Flipping a coin is random if it has no bias.

Summary

Relative Frequency provides an empirical estimate of probability based on actual results.

Expected Frequency is a theoretical prediction based on known probabilities.

Understanding fairness, bias, and randomness is crucial for interpreting results accurately.



C8.3 Probability of Combined Events

1. Sample Space Diagrams

A **sample space** is a list or diagram used to display all possible outcomes. This could be as a list or a table of values. Making a list of all possible outcomes is known as enumeration.

To create a **sample space** diagram we need to think about the possible outcomes of a situation.

For example,

Let's say we were to flip a fair coin and at the same time roll a fair six-sided die.

The sample space for this situation could be written as a list of the possible combinations.

(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)

Calculating Probability:

If you want the probability of getting Heads and a 3:

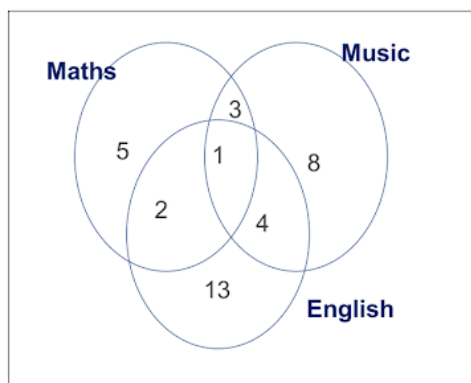
$$P(H,3) = \frac{1(\text{favorable outcome})}{12(\text{total outcomes})} = \frac{1}{12}$$

2. Venn Diagrams

Definition: A Venn diagram visually represents the relationships between two or more sets, showing combined events.

The Venn diagram shows the options that a group of students have chosen for their A-Levels.

A person is selected at random from this group. Given that this person studies music, what is the probability that they study maths.



Example:

Let Set A = people who like ice cream, and Set B = people who like cake.

If 30 like ice cream, 20 like cake, and 10 like both, you can represent this with overlapping circles.

Calculating Probabilities:

$$P(A) = \frac{30}{100} = 0.3$$

$$P(B) = \frac{20}{100} = 0.2$$

$$P(A \cap B) = \frac{10}{100} = 0.1$$

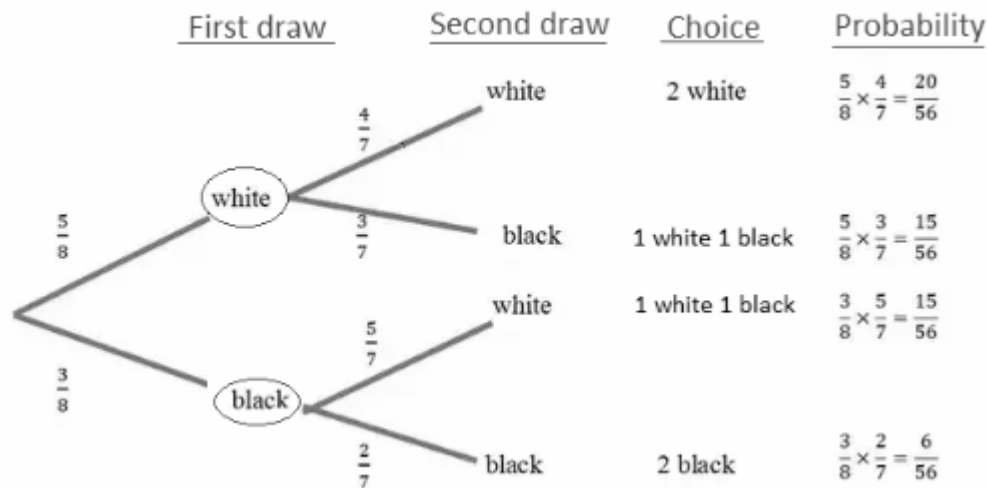
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.2 - 0.1 = 0.4$$

(making venn diagrams by interpreting questions like this is a great way to deepen your understanding of this concept)



3. Tree Diagrams

Definition: A tree diagram displays all possible outcomes of a series of events, showing probabilities along the branches.



Example: If you pick a fruit from a basket containing apples (A) and bananas (B):

First Pick:

A (0.5)

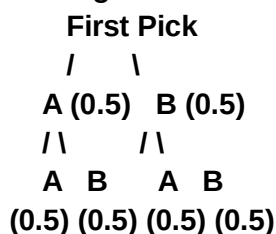
B (0.5)

Second Pick (with replacement):

From A: A (0.5) or B (0.5)

From B: A (0.5) or B (0.5)

Tree Diagram:



Calculating Probabilities:

Probability of picking an Apple both times:

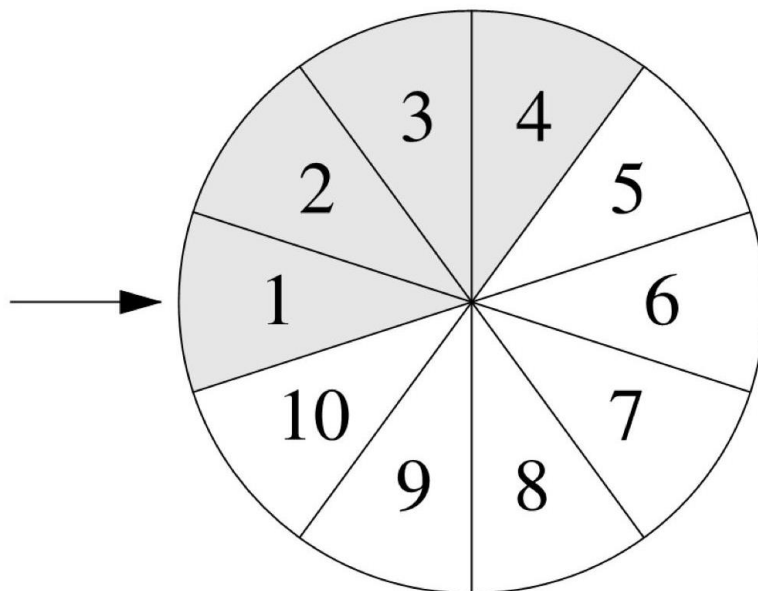
$$P(A,A) = 0.5 \times 0.5 = 0.25$$

Probability of picking an Apple first and a Banana second:

$$P(A,B) = 0.5 \times 0.5 = 0.25$$

(Sample Space Diagrams are useful for visualizing all outcomes. Venn Diagrams are effective for understanding the relationships between two events. Tree Diagrams are helpful for displaying outcomes of sequential events and calculating combined probabilities.)

(Here is an example of a question that you might face in the exams)



A wheel is divided into 10 sectors numbered 1 to 10 as shown in the diagram.

The sectors 1, 2, 3 and 4 are shaded.

The wheel is spun and when it stops the fixed arrow points to one of the sectors.

(Each sector is equally likely.)

(a) The wheel is spun once so that one sector is selected. Find the probability that

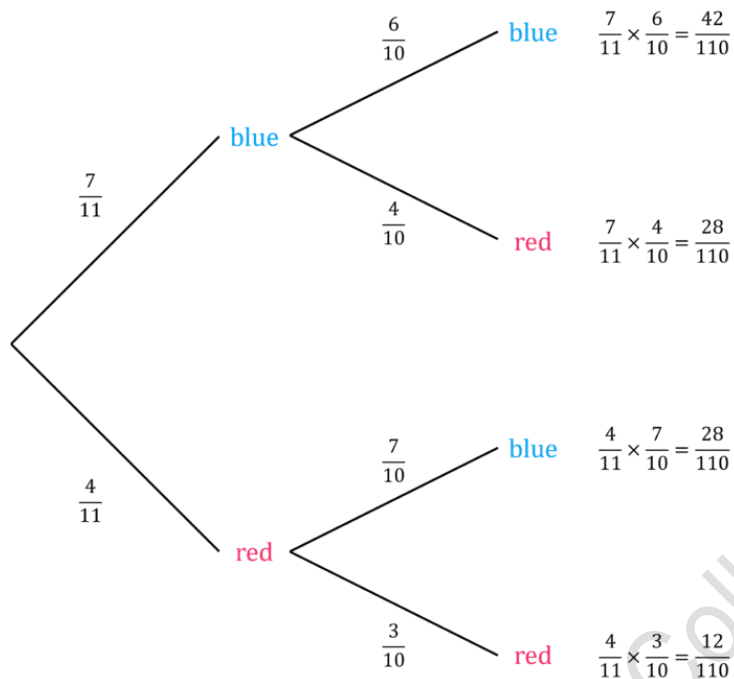
- ✓(i) the number in the sector is even, $\frac{5}{10}$ [1]
- ✓(ii) the sector is shaded, $\frac{4}{10}$ [1]
- ✗(iii) the number is even **or** the sector is shaded, $\frac{7}{10}$ [1]
- ✓(iv) the number is odd **and** the sector is shaded, $\frac{2}{10}$ [1]

(b) The wheel is spun twice so that each time a sector is selected. Find the probability that

- ✓(i) both sectors are shaded, $\frac{16}{100} = \frac{4}{25}$ [2]
- ✗(ii) one sector is shaded and one is not, $\frac{24}{100} = \frac{6}{25}$ [2]
- ✓(iii) the sum of the numbers in the two sectors is greater than 20, 0 [2]
- ✓(iv) the sum of the numbers in the two sectors is less than 4, [2]
- ✓(v) the product of the numbers in the two sectors is a square number. [3]



(Most of the tree diagram questions will be very similar to this, also in a few questions it might be a good habit to make a tree diagram even if it is not required by the question to avoid making silly mistakes and to help understand the question)



(Read the question slowly and carefully as probability is not a particularly hard topic but the questions can be very tricky)

Box 1 contains 2 white balls. Box 2 contains 4 white balls and 3 black balls.

(a) Ann chooses, at random, one ball from each box.

(i) Find the probability that these balls are both black.

Answer 0 [1]

(ii) Find the probability that these balls have different colours.

$\frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{3}{7} = \frac{3}{7}$

Answer $\frac{3}{7}$ [1]

(Do as many past paper questions as you can to internalize these concepts)



UNSOLVED PAST PAPER QUESTIONS:

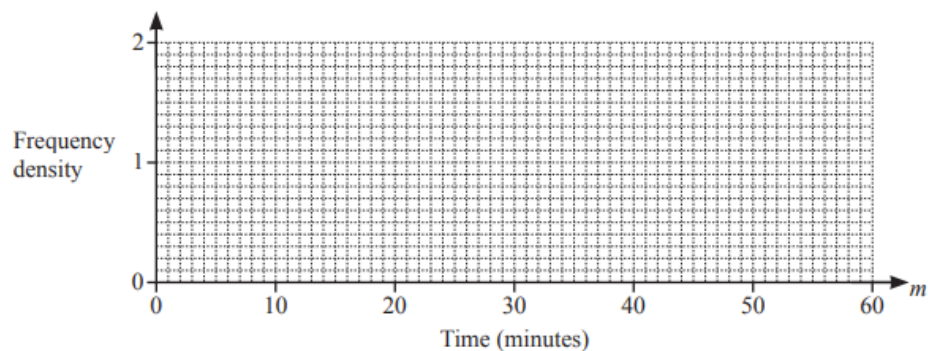
- 3 (a) The table shows the time that each of 40 students takes to travel to school.

Time (m minutes)	$0 < m \leq 10$	$10 < m \leq 25$	$25 < m \leq 40$	$40 < m \leq 60$
Frequency	3	18	15	4

- (i) Calculate an estimate of the mean.

..... min [4]

- (ii) On the grid, draw a histogram to show the information in the table.



[3]

- (iii) Two students are selected at random from the 40 students.

Calculate the probability that one student takes more than 25 minutes and the other student takes 10 minutes or less to travel to school.



ANSWER KEY:

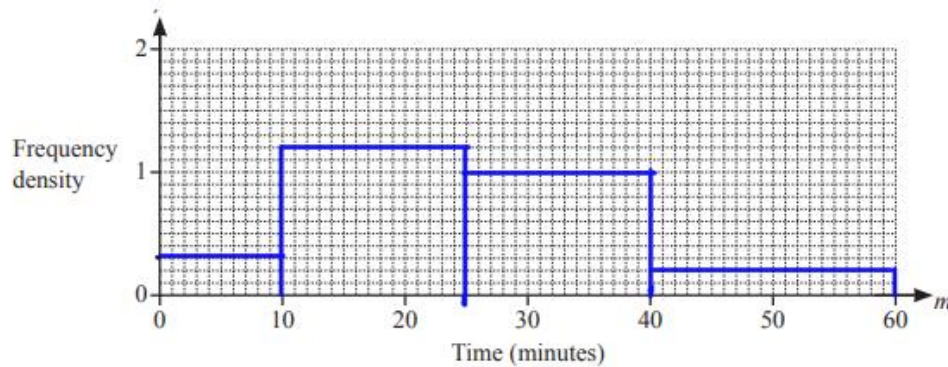
- 3 (a) The table shows the time that each of 40 students takes to travel to school.

Time (m minutes)	$0 < m \leq 10$	$10 < m \leq 25$	$25 < m \leq 40$	$40 < m \leq 60$
Frequency	3	18	15	4

- (i) Calculate an estimate of the mean. $\text{Formula} = \frac{\sum fn}{\sum n}$
- $\sum n = 105$
- $\sum fn = 1017.5$
- $\frac{1017.5}{105} = 9.69$

9.69 min [4]

- (ii) On the grid, draw a histogram to show the information in the table.





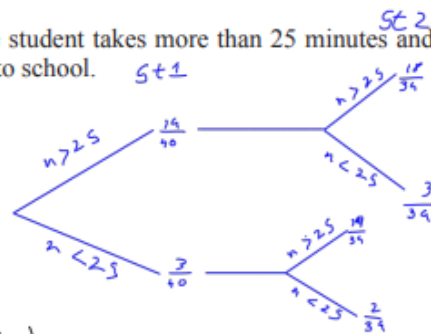
[3]

- (iii) Two students are selected at random from the 40 students.

Calculate the probability that one student takes more than 25 minutes and the other student takes 10 minutes or less to travel to school.

People $< 10 \text{ min} = 3$

People $> 25 \text{ min} = 15 + 4 = 19$



$$P = \left(\frac{19}{40} \times \frac{3}{39} \right) + \left(\frac{3}{40} \times \frac{19}{39} \right)$$

$$= \frac{57}{1560} + \frac{57}{1560}$$

$$= \frac{114}{1560}$$

$$= \frac{19}{260}$$

$$\frac{19}{260} \quad [3]$$



UNIT 9 STATISTICS:

9 Statistics

E9.1 Classifying statistical data

Notes and examples

Classify and tabulate statistical data.

e.g. tally tables, two-way tables.

E9.2 Interpreting statistical data

Notes and examples

- 1 Read, interpret and draw inferences from tables and statistical diagrams.
- 2 Compare sets of data using tables, graphs and statistical measures.
- 3 Appreciate restrictions on drawing conclusions from given data.

e.g. compare averages and measures of spread between two data sets.

E9.3 Averages and measures of spread

Notes and examples

- 1 Calculate the mean, median, mode, quartiles, range and interquartile range for individual data and distinguish between the purposes for which these are used.
- 2 Calculate an estimate of the mean for grouped discrete or grouped continuous data.
- 3 Identify the modal class from a grouped frequency distribution.

E9.4 Statistical charts and diagrams

Notes and examples

Draw and interpret:

- (a) bar charts
- (b) pie charts
- (c) pictograms
- (d) stem-and-leaf diagrams
- (e) simple frequency distributions.

Includes composite (stacked) and dual (side-by-side) bar charts.

Stem-and-leaf diagrams should have ordered data with a key.



9 Statistics (continued)

E9.5 Scatter diagrams

Notes and examples

- 1 Draw and interpret scatter diagrams.
- 2 Understand what is meant by positive, negative and zero correlation.
- 3 Draw by eye, interpret and use a straight line of best fit.

Plotted points should be clearly marked, for example as small crosses (x).

A line of best fit:

- should be a single ruled line drawn by inspection
- should extend across the full data set
- does not need to coincide exactly with any of the points but there should be a roughly even distribution of points either side of the line over its entire length.

E9.6 Cumulative frequency diagrams

Notes and examples

- 1 Draw and interpret cumulative frequency tables and diagrams.
- 2 Estimate and interpret the median, percentiles, quartiles and interquartile range from cumulative frequency diagrams.

Plotted points on a cumulative frequency diagram should be clearly marked, for example as small crosses (x), and be joined with a smooth curve.

E9.7 Histograms

Notes and examples

- 1 Draw and interpret histograms.
- 2 Calculate with frequency density.

On histograms, the vertical axis is labelled 'Frequency density'.
Frequency density is defined as
 $\text{frequency density} = \text{frequency} \div \text{class width}$.



Statistics

1 Classifying Statistical Data

1.1 Classify and tabulate statistical data

This section deals with how to read and interpret tables properly, classifying the dataset as we go forwards. We need to know two main types of tables for this part of the syllabus:

1. Tally Tables
2. Two-way tables

Item	Frequency	Tally			Group A	Group B	Total
Apples	5			Category 1	12	8	20
Oranges	3			Category 2	10	15	25
Bananas	6			Category 3	7	13	20
Grapes	4			Total	29	36	65

The table on the left hand side is called a tally table. The first column tells us what item/object we are talking about. Frequency in the second column refers to the number of occurrences of that item/object. For example a frequency of 5 for the apples means there are 5 apples. The last column is what we call the tally column. Each vertical line is equivalent to 1 tally and we represent the number using these vertical lines. However, as you may have already noticed, this is a very inefficient way of representing data especially if the frequency of the independent variable is very large, for example 100 or 1000.

The table on the right hand side represents a two-way table. Any table would have some kind of categories. It's called a two-way table as it has categories in both the rows and columns. For example in the columns we have groups like A and B and total, and in the rows we have the categories of an object and then the total row. This makes writing data more efficient. A way to interpret such tables is to always keep in mind the row and column you are reading. For example, I ask myself a question:

1. What does the 13 in the table represent?

The answer to that: It lies in the group B column so must be related to Group B and it lies in the Category 13 row so must be related to that so it means either of the following things since they are both the same:

1. The number of category 3 objects that are group B too
2. The number of group B objects that are category 3 objects too

2 Interpreting Statistical Data

2.1 Statistical Methods of Evaluation to compare two or more datasets

2.1.1 Mean/Average

The mean is a mathematical evaluation of the central tendency of the dataset provided. Mathematically in the IGCSE syllabus, it can be defined as the following where X represents a dataset:

P_N

$\sum_{i=1}^N X_i$

$M(X) =$

$N(1)$

$N = X_1 + X_2 + X_3 + \dots + X_N$



What the formula essentially says is that you take each element (number) in the dataset, you add them all together to get the total sum, and then you divide the sum achieved by the total number of elements (numbers) in the dataset. Taking an example to understand this concept:

1. Dataset = {1,2,3,4}

To calculate its mean, we first need the sum of the elements which is $1+2+3+4 = 10$ and then total number of elements is 4 so the mean is:

$$M(X) = \frac{10}{4} = 2.5 \quad (2)$$

2.1.2 Median

In simple terms, the median gives us the middle value of the dataset: in other words the midpoint around which the points in the dataset are scattered around. There are a few rules to calculating the median which can be expressed as follows and must be read with great care:

1. Irrespective of how many number there are in the dataset, the first step is always to arrange the numbers in ascending order (starting from the smallest number and ending with the largest number)
2. Start crossing out one element from the left side and simultaneously, one from the right hand side until you arrive at the middle. If only one value is left at the end, then that's the answer. If there are two values at the end, you take their average $\frac{x_1+x_2}{2}$

where x_1 and x_2 are the two numbers.

There's also another method to calculate the median using a formula. Both would be marked correct in the IGCSE papers. For this method you have to first determine whether there are odd number of elements or even number of elements. Once done, move on the final step for each one as follows:

2

1. If there are an even number of elements, the formula is $\left(\frac{N}{2}\right) + \left(\frac{N+1}{2}\right)$

N is the total number of elements where

2. If there are an odd number of elements, the formula is again the same with the N the number of elements: $\frac{N+1}{2}$

2

2.1.3 Mode

Mode refers to the value that's repeated the most in the dataset. So let's say we have the dataset:

$D = \{1, 2, 3, 4, 4, 4\}$ (3)

4 is repeated the most (a total of 3 times) while all other numbers are just written only once. Therefore, your mode would be 4.

2.1.4 Range

Range refers to the spread of the data, how far away from the midpoint your data is, without considering the variance of the dataset. There's only one formula in the IGCSE syllabus to calculate it:

$$R = Y_{max} - Y_{min} \quad (4)$$

Here Y_{max} is the largest number in the dataset and Y_{min} is the smallest number in the dataset. So if we have the same dataset the range would be:

$$R = 4 - 1 = 3 \quad (5)$$

2.2 Grouped Data - Calculating Mean and Mode

Grouped data is a form of data representation in table-form where instead of separate independent categories, there are certain intervals given for each row. An example can be seen below which will be used for all calculations of this section: The interval for ages is what makes this a grouped data. All the



Table 1: People Age and Frequency Distribution

People Age	Frequency
19 - 29	21
29 - 34	10
34 - 39	22
39 - 44	22
44 - 49	14
49 - 54	11
54 - 60	8
Total	108

examples we previously looked at for mean and mode were un-grouped data.

Now to calculate the mean, follow the steps:

3

1. The formula to calculate mean for grouped data is: $\frac{\sum f_i m_i}{\sum f_i}$. This means:

you are going to calculate the midpoint of each interval and make a separate column of that, then multiply each of those midpoints m with the frequency column respectively to calculate $f_i m_i$ after which sum them all together. This value that you get, divide it by the total of the frequency, which in the Table [1] is 108. To calculate median, the steps are as follows:

1. Find $\sum f$ which is 108 (the total of the frequency) and then apply the even-odd rule for it
2. Then find the cumulative frequency for the dataset by adding each of the consecutive frequencies and adding it in front of the new frequency's row.
3. Next see between in which row does your median lie
4. That row is your answer for the median since for grouped datasets, at IGCSE level, we cannot find the exact median value in it

2.3 Modal Class in a grouped frequency distribution

To calculate the modal class, simply see which distribution range has the highest frequency value - that is your answer.

2.4 Spread of a Data

The spread of a data refers to how spread out from the central point the data is. Let's say the central point is 50 so if we have data that is 49,49,48,51 it's very close to the central point so it's not very spread out. On the other hand, if we had data like 101,1000,2000,300 then we would say it's very spread out - has a greater standard deviation (you don't need to know this term at IGCSE level)

3 Statistical Charts and Diagrams

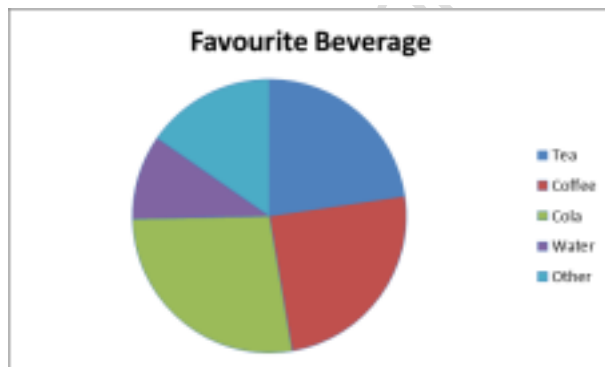
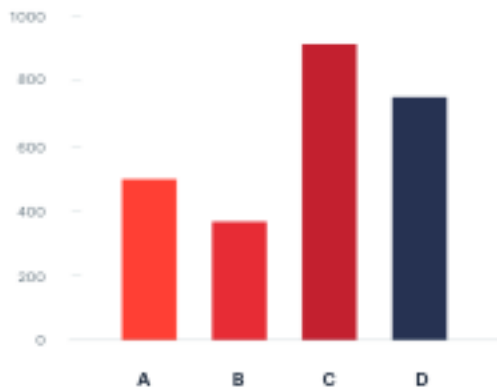
The charts you need to know are all displayed below with their keys to help you understand better.



4 Scatter Diagrams

Scatter diagrams are a type of representation of data that uses scatter points to identify trends. An example can be seen in the figure shown below. A positive correlation means that as x increases, y also increases. Zero correlation means the x and y variables are not related at all - there's no relationship/trend between them. A negative correlation means an inverse relation as one increases,

4



the other decreases. For example, it can be that as y increases, x decreases or vice versa.

You should also know how to make a best fit line. A best fit line is a line that represents the approximate correlation between the two variables graphically - this line should pass through the middle of the entire dataset and should have roughly equal number of points above and below it to ensure it's an accurate representation of the correlation. A best way to draw this is to use your ruler and roughly draw a number of lines and see which one best fits how the data progresses as you move forward on the x-axis, for example. The line of best fit should follow the following rules:

1. It should be a single ruled line drawn by inspection

5



Mode of transport	Number of students
Bus	8 smiley faces
Car	4 smiley faces
Walking	6 smiley faces
Bycycle	3 smiley faces
Key :  Represents 4 children	

Key: 3|1 means 31

Stem	Leaf
1	9 9
2	0 4 7 8
3	1 2 2 2 6
4	0 5 5
5	5

2. It should extend across the full dataset
3. It does not need to coincide exactly with any of the points but there should be a roughly even distribution of points either side of the line over its entire length.

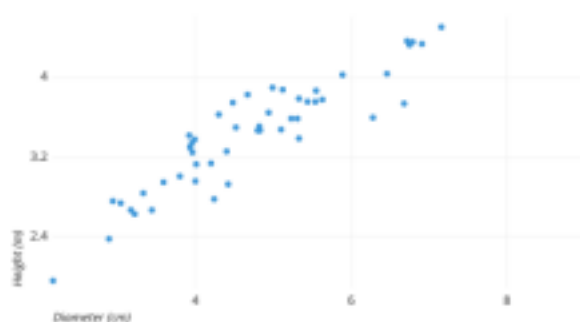
5 Cumulative Frequency Diagrams

As discussed above, cumulative frequency is obtained by adding each of the frequencies. When asked to calculate the median you have to do:

1. See the maximum value on the y-axis and take its half on the y-axis and then draw a line from that point connecting it with the graph
2. Next check the x-axis to see the point at which at that point of intersection lies - that's your answer.

For lower quartile, which means 25%, follow the same process but this time find the mid value between 0 and the median. Similarly, for upper quartile, which

6



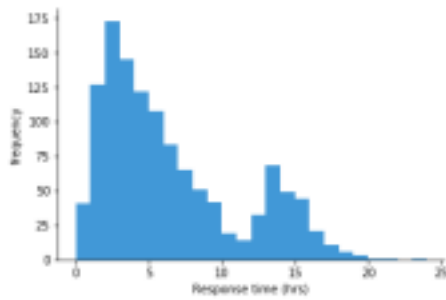


Figure 1: Histogram

is 75%, follow the same process but find the midpoint between the median and the maximum value on the y-axis of the graph. When asked to draw such a graph, always use crosses and join the points with a smooth curve.

6 Histograms

Histograms represent the frequency for each of the range. One important thing about histogram is the frequency density - this would always form the x-axis and is calculated as:

$$D = \frac{f}{W(6)}$$

Where f is the frequency of the range and W is the class width which is the difference between the upper and lower bounds of a class. Finally, here's what a histogram looks.



SOLVED EXAMPLES:

6 The diagram shows a gauge for measuring the water level in a reservoir.
Readings, in metres, taken over a certain period were as follows:
-2.3, -1.6, -0.4, 0.1, -0.5, 0.3, -1.2.

For these readings

(a) find the difference in metres, between the highest and lowest levels,
(b) find the median,
(c) calculate the mean.

(a) Diff = Biggest - Smallest

$$= (0.3) - (-2.3)$$

$$= 0.3 + 2.3 = 2.6$$

(b) $-2.3, -1.6, -1.2, -0.5, -0.4, 0.1, 0.3$
 Median -0.5

(c) Mean = $\frac{-6 + 0.4}{7}$

Answer (a) 2.6 m [1]
 (b) -0.5 m [1]

metres
 3
 2
 1
 0
 -1
 -2
 -3

Negatives
 2.3
 1.6
 1.2
 0.5
 $+ 0.4$
 $\hline 6.0$

Positives
 0.1
 0.3
 0.4

- 8 The heights of 40 children were measured.

The results are summarised in the table below.

Mid(x)	110	120	130	140	
Height (h cm)	$105 < h \leq 115$	$115 < h \leq 125$	$125 < h \leq 135$	$135 < h \leq 145$	
Frequency	5	10	20	5	$\Sigma f = 40$
fx	550	1200	2600	700	$\Sigma fx = 5050$

- (a) (i) Identify the modal class.
 (ii) Calculate an estimate of the mean height.

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{5050}{40} =$$

Answer (a)(i) $125 < h \leq 135$ [1]



UNSOLVED PAST PAPER QUESTIONS:

Here are the heights in centimetres of 20 men.

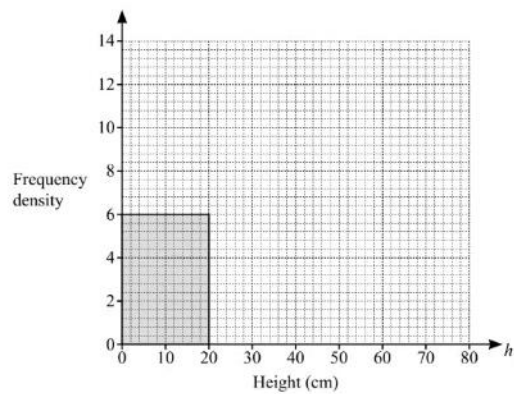
165 164 176 179 188 178 183 172 180 190
167 159 156 176 173 168 169 182 167 192

Show this information in an ordered stem and leaf diagram.

The heights are also shown in the frequency table.

Height (h cm)	$0 < h \leq 20$	$20 < h \leq 30$	$30 < h \leq 40$	$40 < h \leq 80$
Frequency	120	80	124	76

Complete the histogram to show this information.





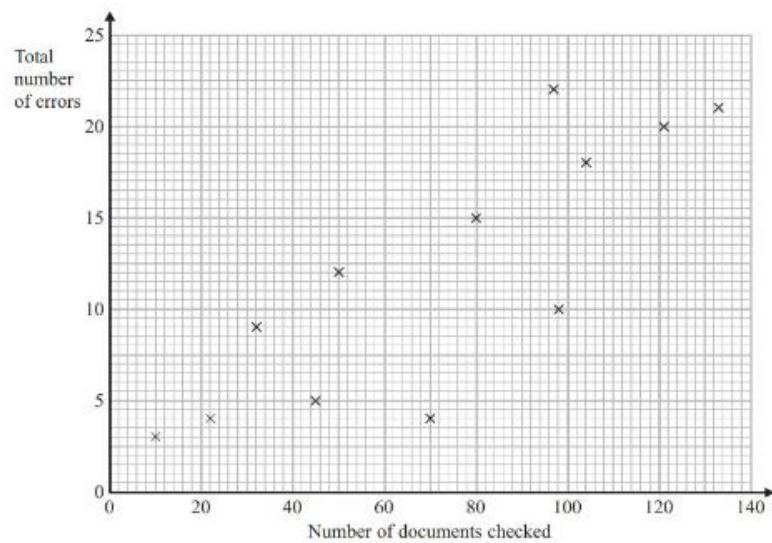
Aitchison

A publisher checks documents for errors.

He records the number of documents that are checked each day.

He also records the total number of errors in the documents each day.

The scatter graph shows this information.



On another day 90 documents are checked.

There is a total of 17 errors.

Show this information on the scatter graph.



ANSWER KEY:

Start by drawing an **unordered** stem-and-leaf diagram.

Cross each number out as you use it and split each value into a stem and a leaf. A leaf will always be the final digit only.

e.g. 165 will have a stem of 16 and a leaf of 5.

As in nature, a stem can have more than one leaf.

Unordered stem-and-leaf diagram

16	5	4	7	8	9	7
17	6	9	8	2	6	3
18	8	3	0	2		
19	0	2				
15	9	6				

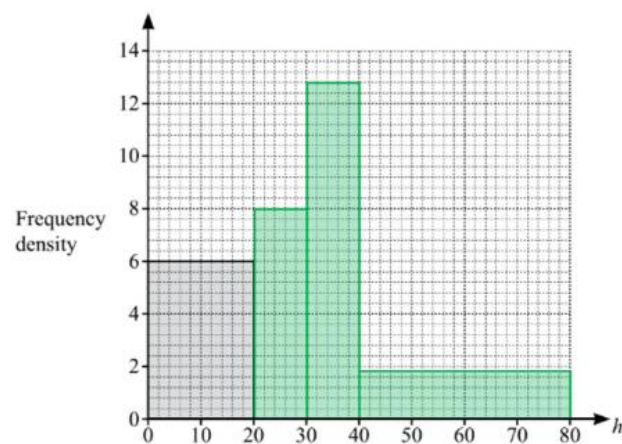
For the final diagram, the stems will need to be put into order, and then their corresponding leaves put into order too.

Keep the leaves in neat columns as this helps to show features of the data such as most frequent group (stem).

Remember to include a **key** with your final answer.

Height (h cm)	$0 < h \leq 20$	$20 < h \leq 30$	$30 < h \leq 40$	$40 < h \leq 80$
Frequency	120	80	124	76
Frequency density	$120 \div 20 = 6$	$80 \div 10 = 8$	$124 \div 10 = 12.4$	$76 \div 40 = 1.9$

Draw the remaining three bars with the correct class width and frequency density. Note that the final bar has a width from 40 to 80





Plot the point (90, 17).

